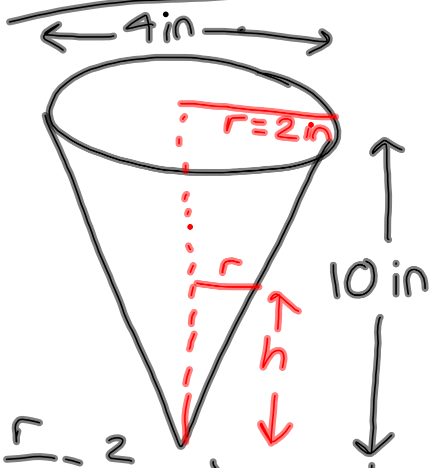
Ice Cream Problem



$\frac{r}{h} = \frac{2}{10} \Rightarrow r = \frac{h}{5}$

$\frac{dV}{dt} = ? \ \frac{in^3}{min}$ when $h = 5in$

$\frac{dh}{dt} = \frac{1in}{5min}$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi\left(\frac{h}{5}\right)^2 \cdot h$$

$$V = \frac{1}{3}\pi \cdot \frac{h^3}{25}$$

$$\frac{dV}{dt} = \frac{1}{\cancel{3}}\pi \cdot \frac{1}{25} \cdot \cancel{3}h^2 \frac{dh}{dt}$$

$$\frac{dV}{dt} = \frac{\pi h^2}{25} \cdot \frac{dh}{dt}$$

$$= \frac{\pi \cancel{(5)^2}}{\cancel{25}} \cdot \frac{1}{5}$$

$$\boxed{\frac{dV}{dt} = \frac{\pi}{5} \ in^3/min}$$

3.2

35. MVT

$$f(x) = \overset{(2-x)^{1/2}}{\sqrt{2-x}}, \quad [-7,2]$$

$$\frac{f(b)-f(a)}{b-a} = \frac{\sqrt{2-2}-\sqrt{2-(-7)}}{2-(-7)} = \frac{0-3}{9} = -\frac{1}{3}$$

$$f'(x) = \frac{1}{2}(2-x)^{-1/2}\cdot(-1)$$

$$\frac{-1}{2\sqrt{2-x}} = -\frac{1}{3}$$

$$2\sqrt{2-x} = 3$$

$$\sqrt{2-x} = \frac{3}{2}$$

$$2-x = \frac{9}{4}$$

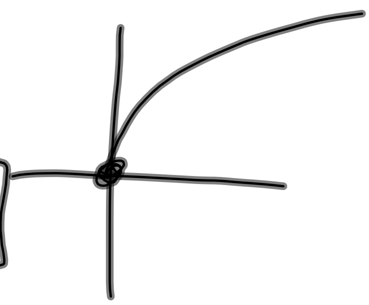
$$\frac{8}{4} - \frac{9}{4} = x$$

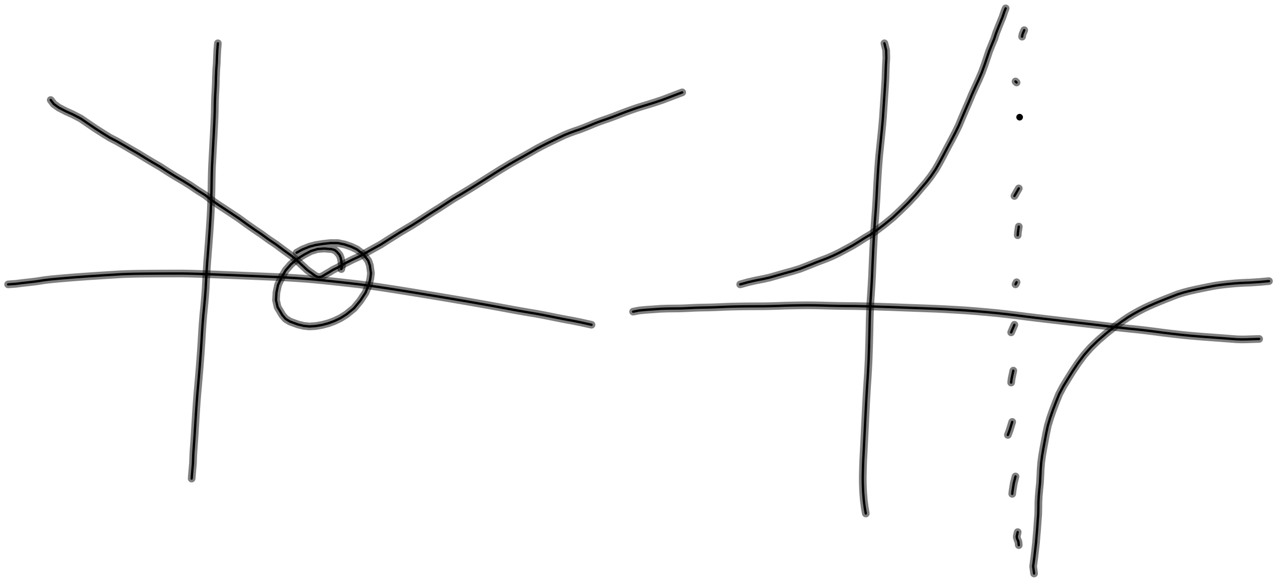
$$\boxed{\frac{-1}{4} = x}$$

[a, b]

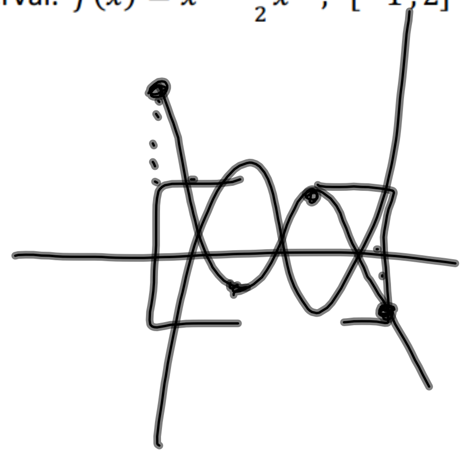
$\sqrt{2-x}$

[-2, 3]





1. Locate the absolute extrema of the function on the closed interval. $f(x) = x^3 - \frac{3}{2}x^2$, $[-1, 2]$



$f'(x) = 3x^2 - 3x$

$3x(x-1) = 0$

critical #'s : 0 & 1

$f(-1) = \frac{-5}{2}$ abs min

$f(0) = 0$

$f(1) = \frac{-1}{2}$

$f(2) = 2$ abs max

2. Determine if Rolle's Theorem can be applied to f on the closed interval [a , b]. If Rolle's Theorem can be applied, find all values of c in the open interval (a, b) such that f'(c)=0.

$$f(x) = (x-3)(x+1)^2 \ , \qquad [-1, 3]$$

f is cts, diff. & f(a) = f(b) ✓

$f(x) = (x-3)(x^2+2x+1)$

$= x^3+2x^2+x-3x^2-6x-3$

$= x^3-x^2-5x-3$

$f'(x) = 3x^2-2x-5$

$3x^2-2x-5 = 0$

$3x^2+3x-5x-5 = 0$

$3x(x+1)-5(x+1) = 0$

$(x+1)(3x-5) = 0$

$\cancel{x=-1}$ $x = 5/3$

not in open interval

3. Determine whether the Mean Value Theorem can be applied to f on the closed interval [a, b]. If the Mean Value Theorem can be applied, find all values of c in the open interval (a, b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$. $f(x) = x(x^2 - x - 2)$, $[-1, 1]$

4. Find the open intervals on which the function is increasing or decreasing and locate all relative extrema. $f(x) = (x+2)^2(x-1)$

5. Find the open intervals on which the function is increasing or decreasing and locate all relative extrema. $y = \frac{x^2}{x^2-9}$

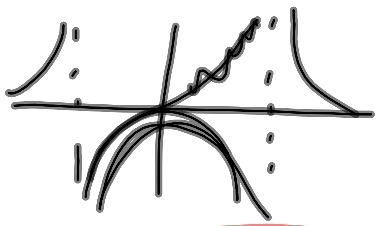
$$y' = \frac{(x^2-9)\cdot 2x - x^2(2x)}{(x^2-9)^2}$$

$$= \frac{\cancel{2x^3} - 18x - \cancel{2x^3}}{(x^2-9)^2} = \frac{-18x}{(x^2-9)^2}$$

critical #'s: $-3, 0, 3$

$f'(-4)$ $f'(-1)$ $f'(1)$ $f'(4)$

$+$ -3 $+$ 0 $-$ 3 $-$



f is increasing on $(-\infty,-3)\cup(-3,0)$

decreasing on $(0,3)\cup(3,\infty)$

relative Maximum @ $(0,0)$

6. Find the points of inflection and discuss concavity of the graph of the function. $f(x) = x^3(x - 4)$

7. Find the points of inflection and discuss concavity of the graph of the function. $f(x) = \frac{x}{x^2+1}$

$$f'(x) = \frac{(x^2+1)\cdot 1 - x(2x)}{(x^2+1)^2} = \frac{-x^2+1}{(x^2+1)^2}$$

$$f''(x) = \frac{(x^2+1)^2\cdot(-2x) - (-x^2+1)\cdot 2(x^2+1)\cdot 2x}{(x^2+1)^4}$$

$$= \frac{\cancel{(x^2+1)}\left(-2x(x^2+1) - 4x(-x^2+1)\right)}{(x^2+1)^{\cancel{4}\,3}}$$

$$= \frac{-2x^3-2x+4x^3-4x}{(x^2+1)^3}$$

$$= \frac{2x^3-6x}{(x^2+1)^3} = \frac{2x(x^2-3)}{(x^2+1)^3}$$

$x^2-3=0$

$x^2=3$

$x=\pm\sqrt{3}$

$f(x)=\frac{x}{x^2+1}$

Inflection points @ $(0, 0)$, $\left(\sqrt{3}, \frac{\sqrt{3}}{4}\right)$ & $\left(-\sqrt{3}, -\frac{\sqrt{3}}{4}\right)$

$f''(-2)$ $f''(-1)$ $f''(1)$ $f''(2)$

$-$ $\quad -\sqrt{3}$ $\quad +$ $\quad 0$ $\quad -$ $\quad \sqrt{3}$ $\quad +$

f is concave up on $(-\sqrt{3}, 0) \cup (\sqrt{3}, \infty)$

down on $(-\infty, -\sqrt{3}) \cup (0, \sqrt{3})$

8. Use the Second Derivative Test to find all relative extrema. $f(x) = x^2 - 6x + 7$

$f'(x) = 2x - 6$

$2(x-3) = 0$

3 is only critical #

$f''(x) = 2 \Rightarrow$ f is always concave up

minimum @ $(3, -2)$

12. The radius of a right circular cylinder is given by $\sqrt{t+2}$ and its height is $\frac{1}{2}t$, where t is time in seconds and the dimensions are in inches. Find the rate of change of the volume with respect to time. Volume of a cylinder is given by $V = \pi r^2 h$, where r is the radius of the cylinder and h is the height.

$r = \sqrt{t+2}$ $\quad \frac{dV}{dt} = ?$ $\quad V = \pi r^2 h$

$h = \frac{1}{2}t$

$$V = \pi\left(\sqrt{t+2}\right)^2 \cdot \frac{1}{2}t$$

$$V = \pi(t+2)\cdot\frac{1}{2}t$$

$$V = \frac{\pi t^2}{2} + \pi t$$

$$\frac{dV}{dt} = \boxed{\pi t + \pi}\ \text{in}^3/\text{s}$$

13. A conical tank is 10 feet across at the top and 10 feet deep. If it is being filled with water at a rate of 5 cubic feet per minute, find the rate of change of the depth of the water when it is 3 feet deep. The volume of a cone is given by $= \frac{1}{3}\pi r^2 h$, where r is the radius of the cone and h is the height. Give an exact answer in terms of π.

14. The radius of a sphere is expanding at a rate of 3 centimeters per second. Find the rate of change of the volume of the cube when the radius is 12 centimeters.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dr}{dt} = 3\,cm/s \; ; \; \frac{dV}{dt} = ? \text{ when } r = 12\,cm$$

$$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$$

$$= 4\pi(12)^2 \cdot 3 = \boxed{12^3\pi \; cm^3/s}$$