Review:

3. Find the limit (if it exists).

$$\lim_{x\to 10}\frac{(2-\sqrt{x-6})}{(10-x)}\cdot\frac{(2+\sqrt{x-6})}{(2+\sqrt{x-6})}$$

$$=\lim_{x\to 10}\frac{4-(x-6)}{(10-x)(2+\sqrt{x-6})}=\lim_{x\to 10}\frac{\cancel{10-x}^{\,1}}{\cancel{(10-x)}(2+\sqrt{x-6})}$$

$$=\frac{1}{2+\sqrt{10-6}}=\frac{1}{2+\sqrt{4}}=\boxed{\frac{1}{4}}$$

3.5 Limits at Infinity

$\lim_{x\to\infty} f(x)$ (end behavior)

correspond exactly with horizontal & oblique asymptotes

$f(x)=\frac{5x^2-3x+4}{2x^2+5x}\approx\frac{5x^2}{2x^2}=\frac{5}{2}$

H.A. @ $y=5/2$

$\lim_{x\to\pm\infty} f(x)=\boxed{\frac{5}{2}}$

$f(x)=\frac{2x-4}{3x^4}\approx\frac{2x}{3x^4}=\frac{2}{3x^3}$

as $x\to\pm\infty$, $f(x)\to 0$

$\lim_{x\to\pm\infty} f(x)=0$

$f(x)=\frac{2x^7-4x^3-2}{5x^4+1}\approx\frac{2x^7}{5x^4}=\frac{2x^3}{5}$

$\lim_{x\to\infty} f(x)=\infty$

$\lim f(x)=-\infty$

$$f(x) = \frac{2 - 7x^3 + 2x}{1+x} \approx -7x^2$$

$$\lim_{x\to\infty} f(x) = -\infty$$

$$\lim_{x\to-\infty} f(x) = -\infty$$

24. $$\lim_{x\to-\infty}\left(\frac{1}{2}x - \frac{4}{x^2}\right) = \boxed{-\infty}$$

$\frac{4}{x^2} \to 0$

$$= \lim_{x\to-\infty} \frac{x^3 - 8}{2x^2} = \lim_{x\to-\infty} \frac{x}{2}$$

$$\approx \frac{x^3}{2x^2}$$

26. $$\lim_{x\to-\infty} \frac{x}{\sqrt{x^2+1}}$$

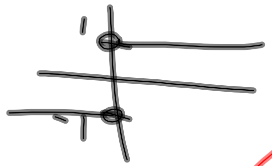
$$= \lim_{x\to-\infty} \frac{x}{\sqrt{x^2}}$$

$$= \lim_{x\to-\infty} \frac{x}{|x|} = \boxed{-1}$$

$$\sqrt[n]{x^n} = \begin{cases} x, & n \text{ odd} \\ |x|, & n \text{ even} \end{cases}$$

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$\sqrt{(-2)^2} = |-2| = 2$$



$$\frac{x}{|x|} = \begin{cases} \frac{x}{x} = 1, & x > 0 \\ \frac{x}{-x} = -1, & x < 0 \end{cases}$$

28. $$\lim_{x\to-\infty} \frac{-3x+1}{\sqrt{x^2+1}}$$

$$\approx \lim_{x\to-\infty} \frac{-3x}{\sqrt{x^2}} = \lim_{x\to-\infty} \frac{-3x}{|x|} = \boxed{3}$$

$$\frac{-3x}{|x|} = \begin{cases} \frac{-3x}{x} = -3, & x > 0 \\ \frac{-3x}{-x} = 3, & x < 0 \end{cases}$$

30. $\displaystyle\lim_{x\to\infty} \frac{x-\cos x}{x}$

$= \displaystyle\lim_{x\to\infty} \frac{x}{x} - \frac{\cos x}{x}$ $\quad \frac{x}{x} \to 1$

$= 1 - \displaystyle\lim_{x\to\infty}\left(\frac{\cos x}{x}\right) \to 0$

$-\frac{1}{x} \le \frac{\cos x}{x} \le \frac{1}{x}$

$-\frac{1}{x} \to 0 \qquad \frac{1}{x} \to 0$

$= \boxed{1}$

32. $\displaystyle\lim_{x\to\infty} \cos\frac{1}{x}$

$= \cos\left[\displaystyle\lim_{x\to\infty}\frac{1}{x}\right]$

$= \cos 0$

$= \boxed{1}$

18. c.

$$\lim_{x\to\infty} \frac{5x^{3/2}}{4\sqrt{x}+1}$$

$$\approx \lim_{x\to\infty} \frac{5x^{3/2}}{4\sqrt{x}} = \lim_{x\to\infty} \frac{5x^{3/2}}{4x^{1/2}}$$

$$= \lim_{x\to\infty} \frac{5x^{2/2}}{4} = \boxed{\infty}$$

7.7 Indeterminate Forms & L'Hôpital's Rule

Indeterminate Forms:

$$\underbrace{\frac{0}{0}, \frac{\infty}{\infty}, \frac{-\infty}{-\infty}, -\frac{\infty}{\infty}}, \ 0\cdot\infty,$$

$$1^{\infty}, 0^{0}, \infty-\infty$$

Let f & g be differentiable functions on an open interval containing c, except maybe at c itself. $g'(x) \neq 0$, except maybe at c.

If $\lim\limits_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{0}{0}$ or $\dfrac{\infty}{\infty}$,

then $\lim\limits_{x \to c} \dfrac{f(x)}{g(x)} = \lim\limits_{x \to c} \dfrac{f'(x)}{g'(x)}$.

7.7

12. $\lim\limits_{x \to -1} \dfrac{x^2 - x - 2}{x + 1} = \lim\limits_{x \to -1} \dfrac{2x - 1}{1}$

$= \dfrac{2(-1) - 1}{1} = \boxed{-3}$

16. $\lim_{x \to 0^+} \frac{e^x - (1+x)}{x^3}$

$= \lim_{x \to 0^+} \frac{e^x - 1}{3x^2} = \lim_{x \to 0^+} \frac{e^x}{6x} = \boxed{\infty}$

$e^x \to 1$

$6x \to 0^+$

18. $\lim_{x \to 1} \frac{\ln x^2}{x^2 - 1}$

$= \lim_{x \to 1} \frac{\frac{1}{x^2} \cdot \cancel{2x}}{\cancel{2x}}$

$= \lim_{x \to 1} \frac{1}{x^2} = \boxed{1}$

20. $\lim_{x\to 0} \frac{\sin ax}{\sin bx}$

$= \lim_{x\to 0} \frac{\sin ax}{ax} \cdot \frac{bx}{\sin bx} \cdot \frac{a}{b}$

$\frac{\sin ax}{ax} \to 1$, $\frac{bx}{\sin bx} \to 1$

$= \lim_{x\to 0} \frac{a\cos ax}{b\cos bx} = \boxed{\frac{a}{b}}$

28. $\lim_{x\to\infty} \frac{x^2}{e^x}$

$= \lim_{x\to\infty} \frac{2x}{e^x}$

$= \lim_{x\to\infty} \frac{2}{e^x} = \boxed{0}$

36. $\lim_{x \to \infty} \frac{e^{x/2}}{x}$

$= \lim_{x \to \infty} \frac{e^{x/2} \cdot \frac{1}{2}}{1} = \boxed{\infty}$

7.7 HW 11-35 odd