Given $xy + y^2 = 3x$,

a. Find y' implicitly in terms of x and y.

b. Find y'' implicitly in terms of x and y.

$$xy' + x'y + 2yy' = 3$$

$$xy' + 2yy' = 3 - y \qquad \boxed{\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}}$$

$$\boxed{y' = \frac{3-y}{x+2y}}$$

$$y'' = \frac{(x+2y)(-y') - (3-y)(1+2y')}{(x+2y)^2}$$

$$y'' = \frac{(x+2y)\left(-\frac{3-y}{x+2y}\right) - (3-y)\left(1+2\cdot\frac{3-y}{x+2y}\right)}{(x+2y)^2}$$

7.7

19. $$\lim_{x\to 0} \frac{\sin 2x}{\sin 3x} = \lim_{x\to 0} \frac{\overset{1}{\cancel{\cos 2x}} \cdot 2}{\underset{1}{\cancel{\cos 3x}} \cdot 3} = \frac{2}{3}$$

17. $$\lim_{x\to 0^+} \frac{e^x - (1+x)}{x^n} = \lim_{x\to 0^+} \frac{e^x - 1}{nx^{n-1}}$$

$$= \lim_{x\to 0^+} \frac{e^x}{(n-1)(n)x^{n-2}} = \boxed{+\infty}, \quad n \geq 3$$

27. $\lim_{x\to\infty} \frac{x^3}{e^{x/2}} = \lim_{x\to\infty} \frac{3x^2}{\frac{1}{2}e^{x/2}}$

$= \lim_{x\to\infty} \frac{6x}{\frac{1}{2}\cdot\frac{1}{2}\cdot e^{x/2}} = \lim_{x\to\infty} \frac{6}{\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot e^{x/2}}$

$\frac{1}{10}, \frac{1}{100}, \frac{1}{100000} = \boxed{0}$

28. $\lim_{x\to\infty} \frac{3x^2-2x+1}{2x^2+3}$

29. $\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}} = \lim_{x\to\infty} \frac{x}{\sqrt{x^2}}$

$= \lim_{x\to\infty} \frac{x}{|x|} = \boxed{1}$

$\frac{x}{|x|} = \begin{cases} \frac{x}{x}, & x>0 \\ \frac{x}{-x}, & x<0 \end{cases}$

L'H

$\lim_{x\to\infty} \frac{1}{\frac{1}{2}(x^2+1)^{-1/2}\cdot 2x}$

$= \lim_{x\to\infty} \frac{2\sqrt{x^2+1}}{2x}$

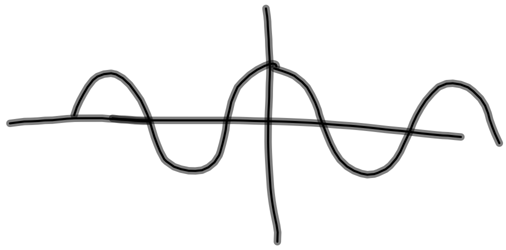
$= \lim_{x\to\infty} \frac{\cancel{2}\cdot\frac{1}{\cancel{2}}(x^2+1)^{-1/2}\cdot\cancel{2}x}{\cancel{2}}$

$= \lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}}$

31. $\lim_{x\to\infty} \frac{\cos x}{x} = \boxed{0}$

$$\frac{-1}{x} \le \frac{\cos x}{x} \le \frac{1}{x}$$

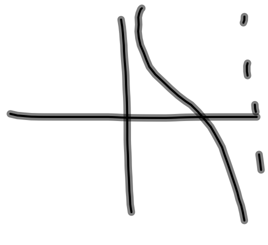
$$\frac{-1}{x} \to 0 \qquad \frac{\cos x}{x} \to 0 \qquad \frac{1}{x} \to 0$$



$0\cdot\infty, \quad 1^{\infty}, \quad 0^{0}$

38. $\lim_{x\to 0^+} x^3 \cot x$

$0\cdot(\infty)$



$$= \lim_{x\to 0^+} \frac{x^3}{\tan x} = \lim_{x\to 0^+} \frac{3x^2}{\sec^2 x} = \boxed{0}$$

44. $\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^x = y$

$$\ln\left[\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x\right] = \ln y$$

$$\lim_{x\to\infty}\left[\ln\left(1+\frac{1}{x}\right)^x\right] = \ln y$$

$$\lim_{x\to\infty}\left[x\ln\left(1+\frac{1}{x}\right)\right] = \ln y$$

$$\lim_{x\to\infty}\frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}} = \ln y$$

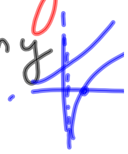
$$\lim_{x\to\infty}\frac{\frac{1}{1+\frac{1}{x}\to 0}\cdot\cancel{\frac{-1}{x^2}}}{\cancel{\frac{-1}{x^2}}} = \ln y$$

$$1 = \ln y$$
$$e^1 = e^{\ln y}$$
$$e = y$$

$$1 = \ln y$$
$$1 = \log_e y$$
$$e^1 = e^{\log_e y}$$
$$e = y$$

50. $\lim\limits_{x\to0^+}\left[\cos\left(\frac{\pi}{2}-x\right)\right]^x = y$

$$\lim_{x\to0^+}\ln\left[\cos\left(\frac{\pi}{2}-x\right)\right]^x = \ln y$$



$$\lim_{x\to0^+} x\ln\left(\cos\left(\frac{\pi}{2}-x\right)\right) = \ln y$$

$$\lim_{x\to0^+}\frac{\ln\left(\cos\left(\frac{\pi}{2}-x\right)\right)}{\frac{1}{x}} = \ln y$$

$$\lim_{x\to0^+}\frac{\frac{1}{\cos\left(\frac{\pi}{2}-x\right)}\cdot\left[-\sin\left(\frac{\pi}{2}-x\right)\right](-1)}{\frac{-1}{x^2}} = \ln y$$

$$\lim_{x\to0^+}\frac{\frac{\sin\left(\frac{\pi}{2}-x\right)}{\cos\left(\frac{\pi}{2}-x\right)}}{\frac{-1}{x^2}} = \ln y$$

$$\lim_{x\to0^+}\frac{\tan\left(\frac{\pi}{2}-x\right)}{\frac{-1}{x^2}} = \ln y$$

$$(-x^{-2})' = 2x^{-3}$$

$$\lim_{x\to0^+}\frac{\sec^2\left(\frac{\pi}{2}-x\right)(-1)}{\frac{2}{x^3}} = \ln y$$

$$\lim_{x\to0^+}\frac{-x^3}{2}\cdot\sec^2\left(\frac{\pi}{2}-x\right) = \ln y$$

$$0 = \ln y$$

7.7

37 – 53 odd