homework questions?

- 1.2 #1-7odd,9-18all
- 1.2 #23, 25, 27, 29, 30, 31 epsilon-delta
- 1.3 #11,17,27-35odd, 39-61odd

<-- note to self, check hw and assign hw grade!

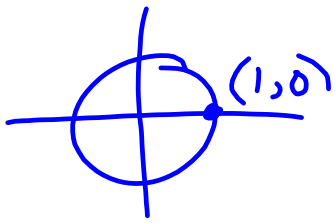
1.3

53. $\lim_{x\to 0} \frac{(\sqrt{x+5}-\sqrt{5})}{x} \cdot \frac{(\sqrt{x+5}+\sqrt{5})}{\sqrt{x+5}+\sqrt{5}} =$

$= \lim_{x\to 0} \frac{x+\cancel{5}-\cancel{5}}{x(\sqrt{x+5}+\sqrt{5})} = \lim_{x\to 0} \frac{1}{\sqrt{x+5}+\sqrt{5}} = \boxed{\frac{1}{2\sqrt{5}}}$

31. $\lim_{x\to 0} \sec 2x = \sec(2\cdot 0) = \sec 0$

$= \frac{1}{\cos 0} = \frac{1}{1} = \boxed{1}$



1.3 Evaluating Limits Analytically

42. $h(x) = \dfrac{x^2 - 3x}{x}$

(a) $\lim\limits_{x \to -2} h(x) = \dfrac{(-2)^2 - 3(-2)}{-2} = \dfrac{4+6}{-2} = \boxed{-5}$

(b) $\lim\limits_{x \to 0} h(x) = \lim\limits_{x \to 0} \dfrac{\cancel{x}(x-3)}{\cancel{x}} = \boxed{-3}$

44. $\lim\limits_{x \to 1} \dfrac{x}{x^2 - x} = \lim\limits_{x \to 1} \dfrac{x}{x(x-1)}$ undefined

48. $\lim_{x \to -1} \frac{x^3 + 1}{x + 1}$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$= \lim_{x \to -1} \frac{\cancel{(x+1)}(x^2 - x + 1)}{\cancel{x+1}}$

$= (-1)^2 - (-1) + 1$

$= 1 + 1 + 1$

$= \boxed{3}$

54. $\lim_{x \to 0} \frac{\sqrt{2+x} - \sqrt{2}}{x} \cdot \frac{\sqrt{2+x} + \sqrt{2}}{\sqrt{2+x} + \sqrt{2}}$

$= \lim_{x \to 0} \frac{\cancel{2} + \overset{1}{\cancel{x}} - \cancel{2}}{\cancel{x}(\sqrt{2+x} + \sqrt{2})} = \boxed{\frac{1}{2\sqrt{2}}}$

58. $\lim_{x\to 0} \dfrac{\frac{4}{4}\cdot\frac{1}{x+4} - \frac{1}{4}\cdot\frac{x+4}{x+4}}{x}$

$= \lim_{x\to 0} \dfrac{\frac{4}{4(x+4)} - \frac{x+4}{4(x+4)}}{x} = \lim_{x\to 0} \dfrac{\left(\frac{4-x-4}{4(x+4)}\right)}{\frac{x}{1}}$

$= \lim_{x\to 0} \dfrac{\frac{-1}{\cancel{x}}}{4(x+4)} \cdot \dfrac{1}{\cancel{x}} = \lim_{x\to 0} \dfrac{-1}{4(x+4)}$

$= \boxed{\dfrac{-1}{16}}$

66. $\lim_{x\to 2} \dfrac{x^5 - 32}{x - 2}$

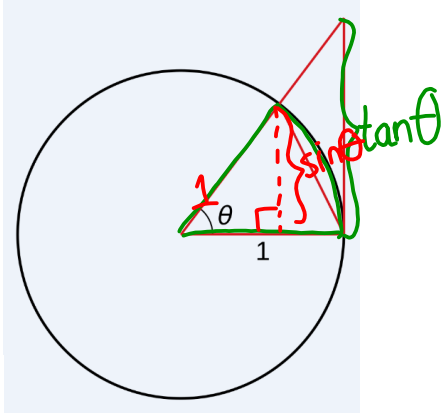
$(a+b)^n \neq a^n + b^n$

2	1	0	0	0	0	-32
		2	4	8	16	32
	1	2	4	8	16	$\boxed{0}$
	x^4	x^3	x^2	x	c	

$= \lim_{x\to 2} \dfrac{\cancel{(x-2)}(x^4+2x^3+4x^2+8x+16)}{\cancel{x-2}}$

$= \boxed{80}$

1.3 The Squeeze Theorem



$$\lim_{x\to 0}\frac{\sin x}{x}=?$$

Area of whole circle $= \pi r^2|_{r=1} = \pi$

$$\frac{\text{Area of whole circle}}{\text{Total angle of circle}} = \frac{\text{Area of sector}}{\theta}$$

$$\frac{\pi}{2\pi} = \frac{\text{Area of sector}}{\theta} \rightarrow \text{Area of sector} = \frac{\theta}{2}$$

Area of outer triangle ≥ Area of sector ≥ Area of inner triangle

$$\frac{\tan\theta}{2} \geq \frac{\theta}{2} \geq \frac{\sin\theta}{2}$$

Multiply through by $\frac{2}{\sin\theta}$

$$\frac{\sin\theta}{2\cos\theta}\cdot\frac{2}{\sin\theta} \geq \frac{\theta}{2}\cdot\frac{2}{\sin\theta} \geq \frac{\sin\theta}{2}\cdot\frac{2}{\sin\theta}$$

$$\frac{1}{\cos\theta} \geq \frac{\theta}{\sin\theta} \geq 1$$

Take reciprocals and reverse inequalities

$$\cos\theta \leq \frac{\sin\theta}{\theta} \leq 1$$

Take limits

$$\lim_{\theta\to 0}\cos\theta \leq \lim_{\theta\to 0}\frac{\sin\theta}{\theta} \leq \lim_{\theta\to 0} 1$$

$$1 \leq \lim_{\theta\to 0}\frac{\sin\theta}{\theta} \leq 1$$

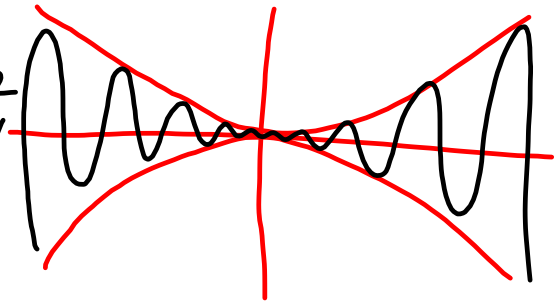
$$\Rightarrow \boxed{\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1}$$

The Squeeze Theorem:

If $f(x) \le g(x) \le h(x)$ *and* $\lim_{x\to c} f(x) = L = \lim_{x\to c} h(x)$,

Then $\lim_{x\to c} g(x) = L$.

$$-x^2 \le x^2 \sin x \le x^2$$



Special Limits Derived by Squeeze Theorem:

$$\boxed{\lim_{x\to 0} \frac{\sin x}{x} = 1} \; ; \; \boxed{\lim_{x\to 0} \frac{1-\cos x}{x} = 0}$$

Memorize!!

Use the squeeze theorem to find

$$\lim_{x\to 0}\left(x^2 \cos\frac{5}{x} - 3\right)$$

$$-1 \le \cos\theta \le 1$$

$$-1 \le \cos\frac{5}{x} \le 1$$

$$-x^2 \le x^2\cos\frac{5}{x} \le x^2$$

$$-x^2 - 3 \le x^2\cos\frac{5}{x} - 3 \le x^2 - 3$$

$$\underbrace{\lim_{x\to 0}(-x^2-3)}_{=-3} \le \lim_{x\to 0}\left(x^2\cos\frac{5}{x} - 3\right) \le \underbrace{\lim_{x\to 0}(x^2-3)}_{-3}$$

By the Squeeze Theorem,

$$\lim_{x\to 0}\left(x^2\cos\frac{5}{x} - 3\right) = \boxed{-3}$$

68. $\lim_{x\to 0} \frac{3(1-\cos x)}{x}$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x\to 0} \frac{1-\cos x}{x} = 0$$

$$= \left(\lim_{x\to 0} (3)\right) \cdot \left(\lim_{x\to 0} \left(\frac{1-\cos x}{x}\right)\right)$$

$$= 3 \cdot 0$$

$$= \boxed{0}$$

72. $\lim_{x\to 0} \frac{\tan^2 x}{x} = \lim_{x\to 0} \frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{x}{1}} = \lim_{x\to 0} \frac{\sin^2 x}{x\cos^2 x} =$

$$= \lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{\sin x}{\cos^2 x}$$

$$= \left(\lim_{x\to 0} \frac{\sin x}{x}\right) \cdot \left(\lim_{x\to 0} \frac{\sin x}{\cos^2 x}\right)$$

$$= 1 \cdot 0$$

$$= \boxed{0}$$

78. $\lim_{x\to 0} \frac{\sin 2x}{\sin 3x}$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\lim_{2x\to 0} \frac{\sin 2x}{2x} = 1$$

$$= \lim_{x\to 0} \frac{\sin 2x}{2x} \cdot \frac{3x}{\sin 3x} \cdot \frac{2}{3}$$

$$= \left(\lim_{x\to 0} \frac{\sin 2x}{2x}\right) \cdot \lim_{x\to 0}\left(\frac{3x}{\sin 3x}\right) \cdot \lim_{x\to 0}\left(\frac{2}{3}\right)$$

$$= 1 \cdot 1 \cdot \frac{2}{3}$$

$$= \boxed{\frac{2}{3}}$$

Homework:

Already assigned:

- 1.2 #1-7odd,9-18all
- 1.2 #23, 25, 27, 29, 30, 31 epsilon-delta
- 1.3 #11,17,27-35odd, 39-61odd

New: 1.3 #67-77odd; 87, 88 (<-- not listed on your syllabus!)

Quiz #1 - Thursday?