Homework questions? §1.3

61. $\lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - 2(x+\Delta x) + 1 - (x^2 - 2x + 1)}{\Delta x}$

$\lim_{h \to 0} \frac{(x+h)^2 - 2(x+h) + 1 - (x^2 - 2x + 1)}{h}$

[limit of difference quotient for $f(x) = x^2 - 2x + 1$]

$= \lim_{\Delta x \to 0} \frac{\cancel{x^2} + 2x\Delta x + (\Delta x)^2 \cancel{-2x} - 2\Delta x \cancel{+1} \cancel{-x^2} \cancel{+2x} \cancel{-1}}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(2x + \Delta x - 2)}{\cancel{\Delta x}} = \boxed{2x - 2}$

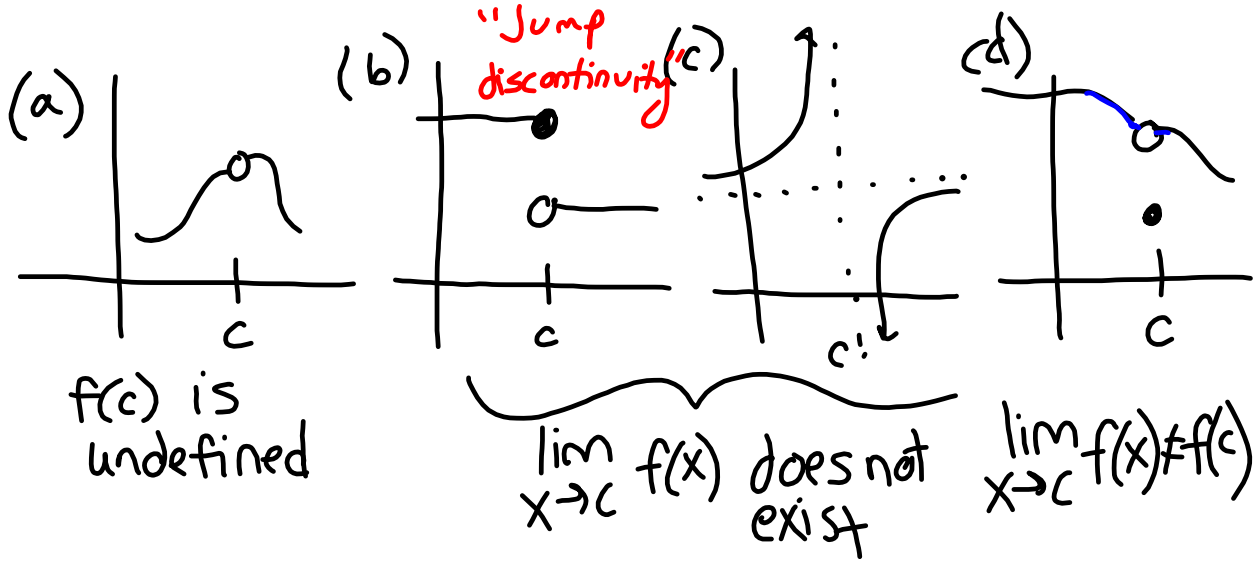
1.3

53. $\lim_{x \to 0} \frac{\sqrt{x+5} - \sqrt{5}}{x} \cdot \frac{\sqrt{x+5} + \sqrt{5}}{\sqrt{x+5} + \sqrt{5}} =$

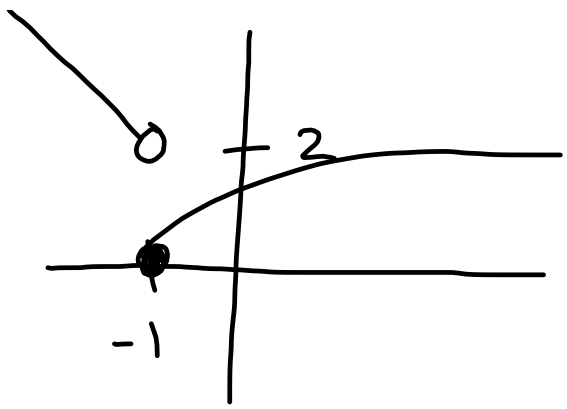
$= \lim_{x \to 0} \frac{x + 5 - 5}{x(\sqrt{x+5} + \sqrt{5})} = \lim_{x \to 0} \frac{1}{\sqrt{x+5} + \sqrt{5}} = \boxed{\frac{1}{2\sqrt{5}}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{\sqrt{5}}{10}$

1.4 Connuity and One-Sided Limits



These are all discontinuities
(a) and (d) are removable
(b) and (c) are nonremovable



$$\lim_{x\to -1^-} f(x) = 2$$

$$\lim_{x\to -1^+} f(x) = 0$$

$$\lim_{x\to -1} f(x) = \text{does not exist}$$

One-Sided Limits

$\lim_{x\to c^+} f(x) = L$ *limit from the right*

$\lim_{x\to c^-} f(x) = L$ *limit from the left*

$$\lim_{x\to c} f(x) = L \text{ if and only if}$$

$$\lim_{x\to c^-} f(x) = L = \lim_{x\to c^+} f(x)$$

Continuity at a point

A function f is continuous at c if the following 3 conditions are met:

1. $f(c)$ is defined
2. Limit of $f(x)$ exists when x approaches c
3. Limit of $f(x)$ when x approaches c is equal to $f(c)$

$f(x)$ is continuous at c if

$$\lim_{x \to c} f(x) = f(c)$$

Continuity on an open interval (a,b)

A function is continuous on an open interval if it is continuous at each point in the interval. A function that is continuous on the entire real line $(-\infty, \infty)$is everywhere continuous.

Continuity on a closed interval $[a,b]$

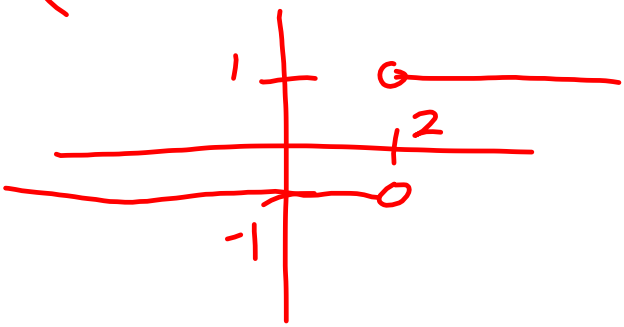
A function f is continuous on the closed interval $[a, b]$ if it is continuous on the open interval I (a, b) and $\lim_{x \to a^+} f(x) = f(a)$ and $\lim_{x \to b^-} f(x) = f(b)$.

10. $$\lim_{x \to 4^-} \frac{\sqrt{x} - 2}{x - 4} \cdot \frac{\sqrt{x} + 2}{\sqrt{x} + 2} =$$

$$= \lim_{x \to 4^-} \frac{x - 4}{(x-4)(\sqrt{x}+2)} = \lim_{x \to 4^-} \frac{1}{\sqrt{x} + 2} = \boxed{\frac{1}{4}}$$

12. $\lim_{x \to 2^+} \frac{|x-2|}{x-2} = \boxed{1}$

$$\frac{|x-2|}{x-2} = \begin{cases} \frac{x-2}{x-2} = 1, & x-2>0 \\ & x>2 \\ \frac{-(x-2)}{x-2} = -1, & x-2<0 \\ & x<2 \end{cases}$$



1.4

Discuss the [dis]continuity of the function.

$$f(x) = \frac{(x+4)(x-2)}{(x-2)(x+1)} = \frac{x+4}{x+1}, \; x \neq 2$$

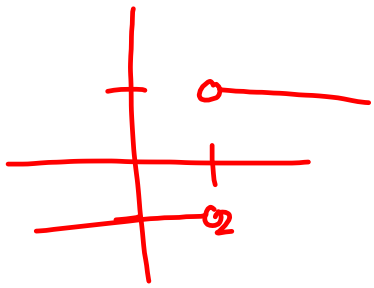
removable discontinuity @ $x=2$ (hole)

non-removable discontinuity @ $x=-1$ (vertical asymptote)

f is continuous on:

$(-\infty,-1) \cup (-1,2) \cup (2,\infty)$

$$f(x) = \frac{|x-2|}{x-2}$$

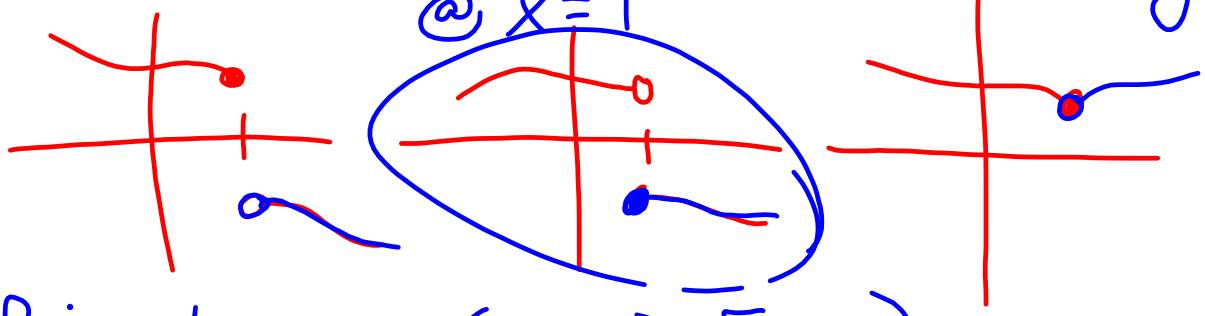


non-removable discontinuity @ $x=2$

f is continuous on $(-\infty, 2) \cup (2, \infty)$

$$f(x) = \begin{cases} x^2-2, & x \geq 1 \\ 5, & x < 1 \end{cases}$$

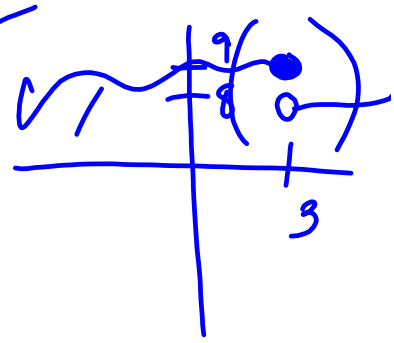
$-1 \neq 5 \Rightarrow$ non-removable discontinuity @ $x=1$



f is cts on $(-\infty, 1) \cup [1, \infty)$

$$f(x)=\begin{cases} x+6, & x\le -2 \\ x^2, & -2<x\le 3 \\ 8 & x>3 \end{cases}$$

non-removable discontinuity @ $x=3$



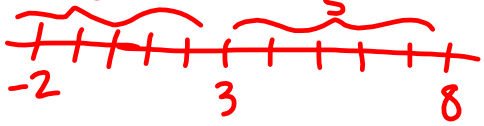
f is cts on

$(-\infty,3]\cup(3,\infty)$

$$f(x)=\begin{cases} \dfrac{|x-3|}{3-x}, & |x-3|>5 \\ x^2-3, & -2\le x\le 8 \end{cases}$$

$x<-2$
$x>8$

$x-3>5$ or $x-3<-5$
$x>8$ or $x<-2$



$$\frac{|x-3|}{3-x}=\begin{cases} \dfrac{x-3}{3-x}=-1, & x-3>0 \\ & x>3 \\ \dfrac{-(x-3)}{3-x}=1, & x-3<0 \\ & x<3 \end{cases}$$

$$f(x)=\begin{cases} 1, & x<-2 \\ x^2-3, & -2\le x\le 8 \\ -1, & x>8 \end{cases}$$

non-removable discontinuity @ $x=8$

f is cts on $(-\infty,8]\cup(8,\infty)$

Homework for Test #1:

HW#1 (submitted Mon. 11/12)

- 1.2 #1-7odd,9-18all
- 1.2 #23, 25, 27, 29, 30, 31 epsilon-delta
- 1.3 #11,17,27-35odd,

HW#2 (due Mon. 11/18?)

- 1.3 #39-61odd (<-- not listed on your syllabus!)
- 1.3 #67-77odd; 87, 88 (<-- not listed on your syllabus!)
- **New: 1.4 #7-17odd; 25-28all; 39-47odd; 57, 59**
- **1.4 #19,21,23,51,63,69,71,83,85**
- **Test #1 Practice Problems** (on web; will hand out hard copy Thurs. in class)

Quiz #1 - Take-home; due Friday 11/15

Test #1 - Wednesday, 11/20