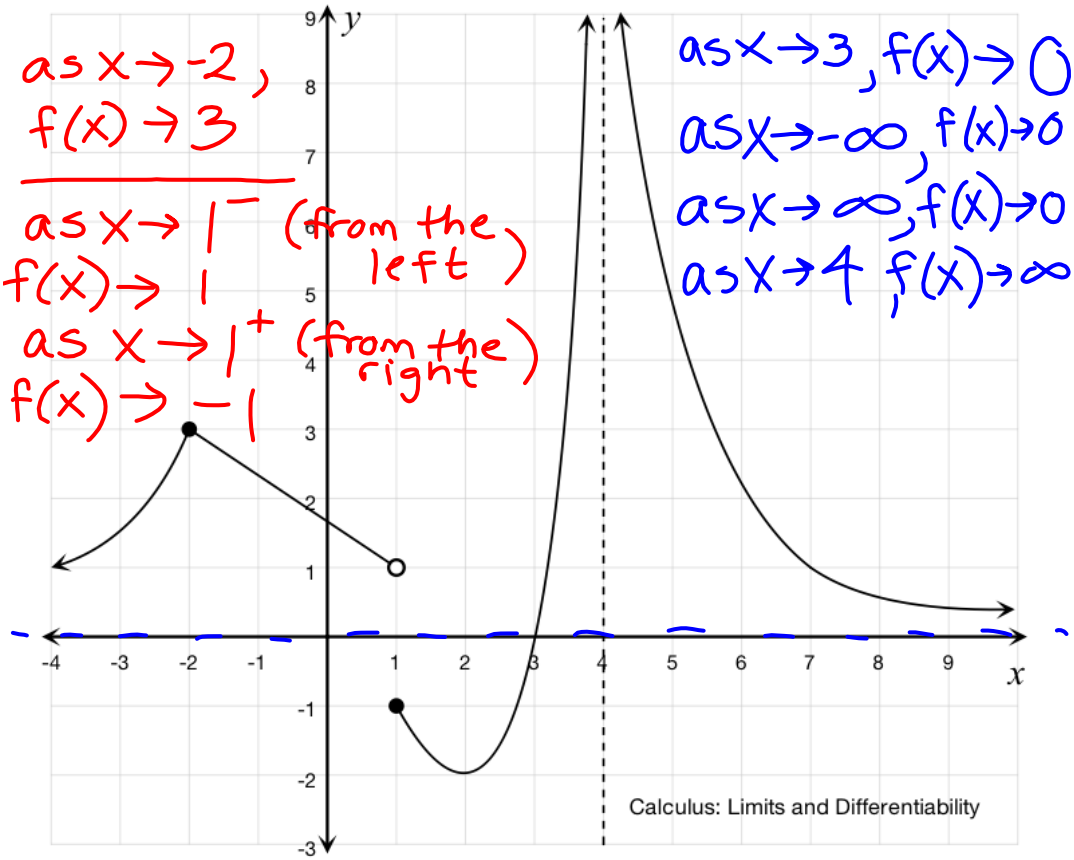
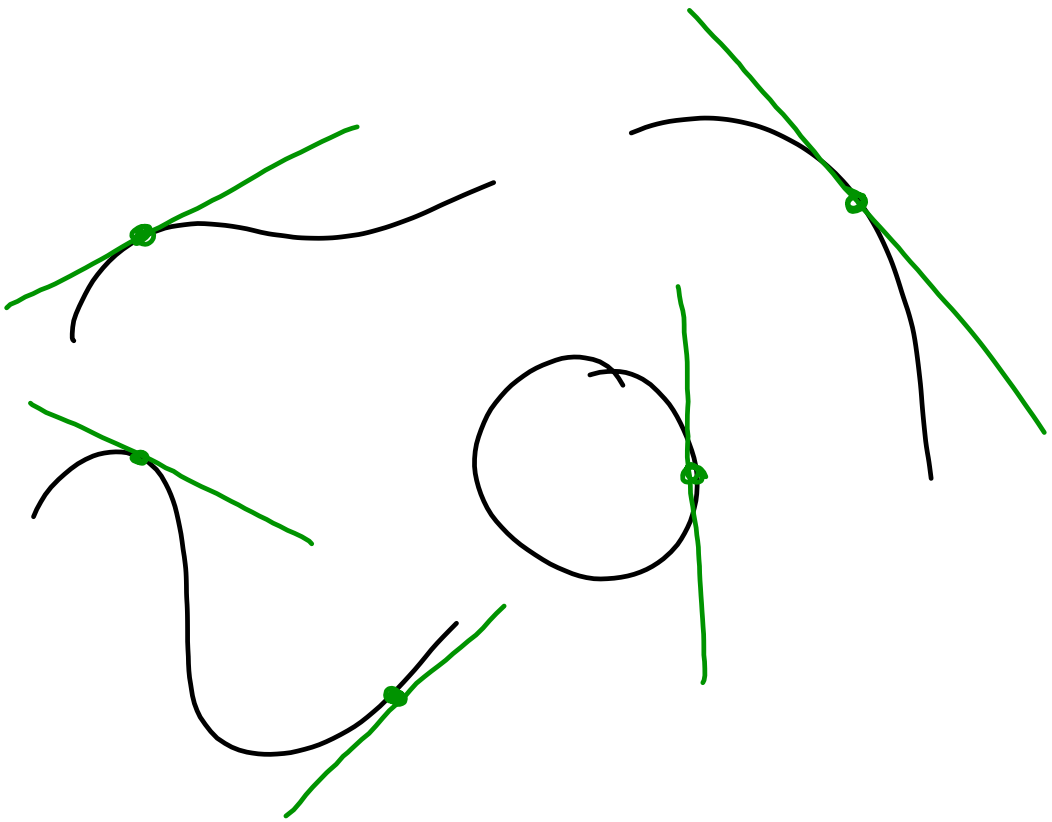
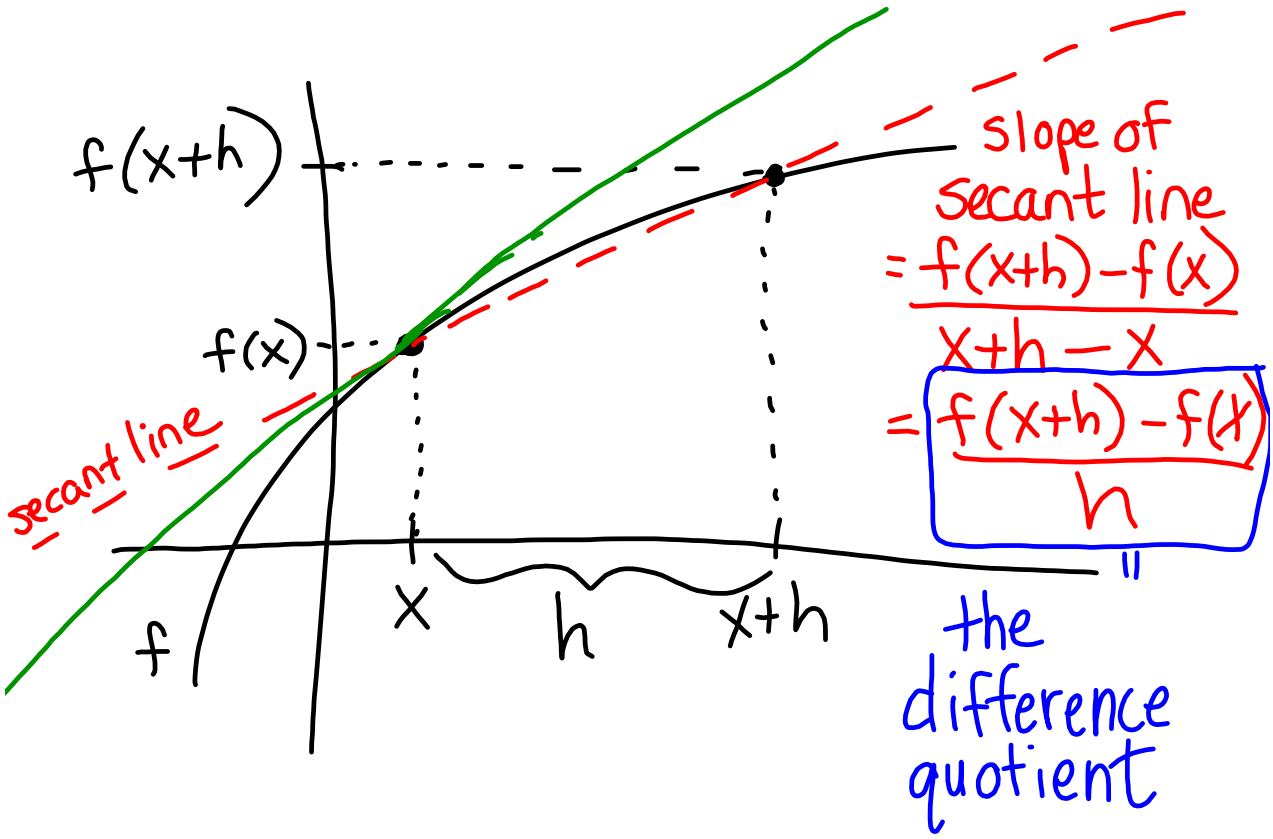
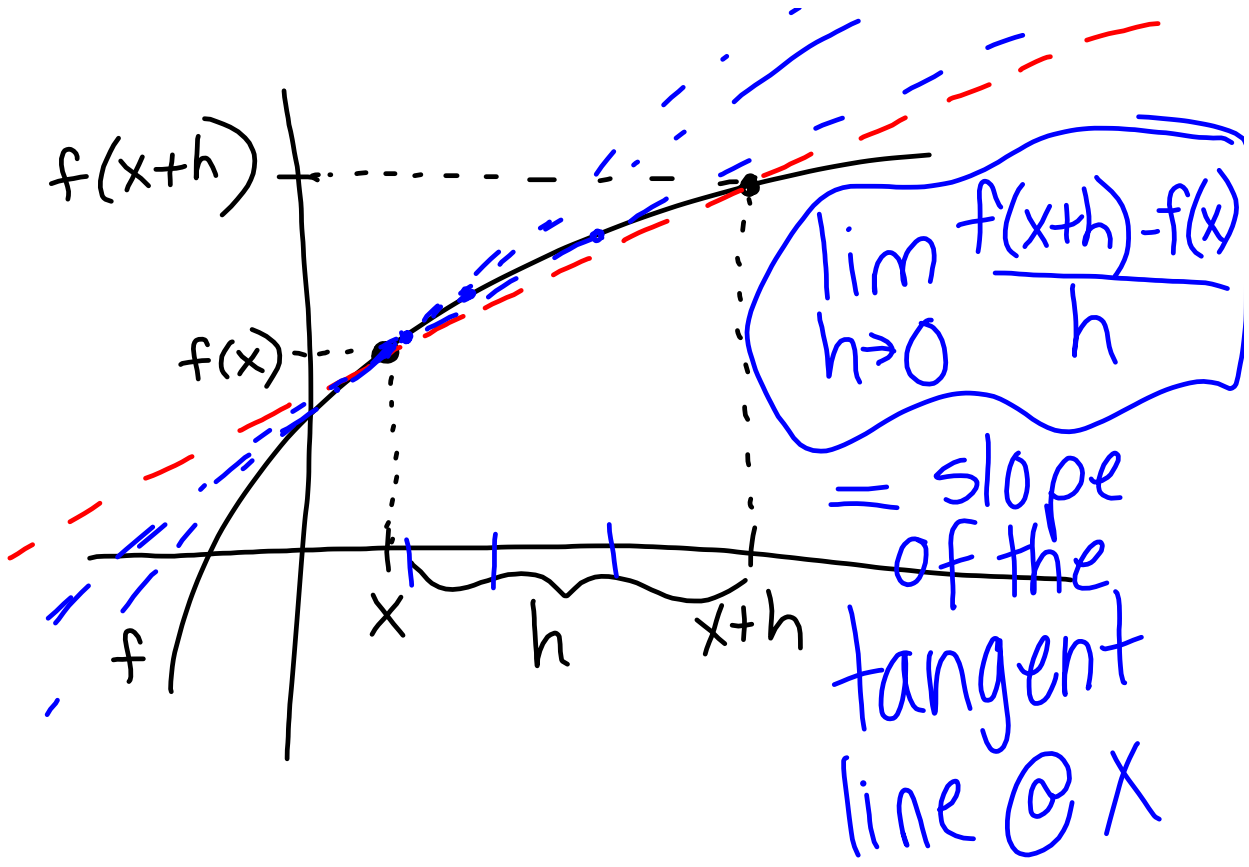


tangent lines




f(x+h)
f(x)
secant line
f
x
h
x+h
slope of secant line
$= \frac{f(x+h)-f(x)}{x+h-x}$
$= \frac{f(x+h)-f(x)}{h}$
the difference quotient


f(x+h)
f(x)
f
x
h
x+h
$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$
= slope of the tangent line @ X

Δx "delta x"

change in x

$$\frac{f(x+\Delta x)-f(x)}{\Delta x} = \frac{f(x+h)-f(x)}{h}$$

Δx ↑ h ↑ Δx

1.2

$$f(x)=\frac{x-2}{x^2-4}, \quad x\neq 2,-2$$

what happens to $f(x)$ as x approaches 2?

$x = 2.01$, $f(x) = 0.24938$ ↓

x	1.9	1.99	1.999	2	2.001	2.1
$f(x)$	0.256 ↑	0.2506	0.2501	$\frac{1}{4}$	0.2499	0.2439

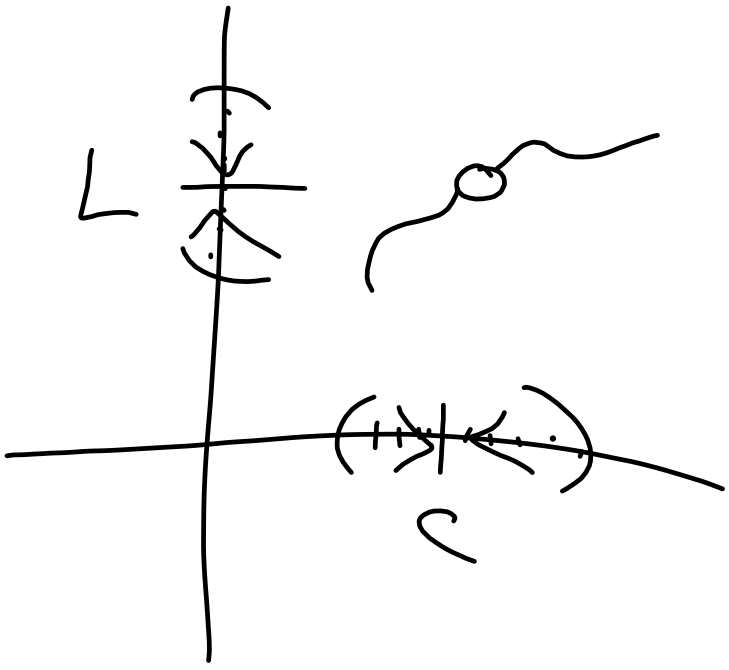
Informal Decription of the Limit

If $f(x)$ becomes arbitrarily close to a single number L as x approaches c from either side, the limit of $f(x)$, as x approaches c, is L.

$$\lim_{x \to c} f(x) = L$$

Note: the existence or nonexistence of $f(x)$ at $x=c$ has no bearing on the existence of the limit as $x \to c$.

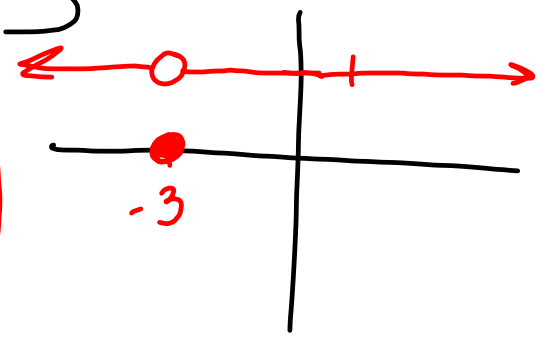
$$\lim_{x \to -3} \frac{\sqrt{1-x} - 2}{x+3} = -0.25$$



$$f(x)=\begin{cases}1, & x\neq -3\\ 0, & x=-3\end{cases}$$

$y=1$

$f(-3)=0$



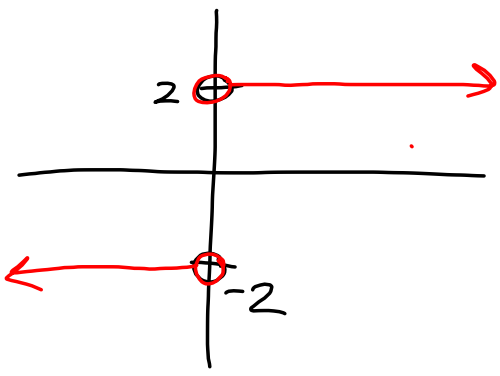
$$\lim_{x\to -3} f(x) = 1$$

The function value is not necessarily equal to the limit

$$\lim_{x\to 0}\frac{|2x|}{x}$$

$$|x|=\begin{cases}x, & x\geq 0\\ -x, & x<0\end{cases}$$

$$\frac{|2x|}{x}=\begin{cases}\frac{2x}{x}=2, & x>0\\ \frac{-2x}{x}=-2, & x<0\end{cases}$$



$\lim_{x\to 0} f(x) = ?$ is undefined

$\lim_{x\to 0^+} f(x) = 2$

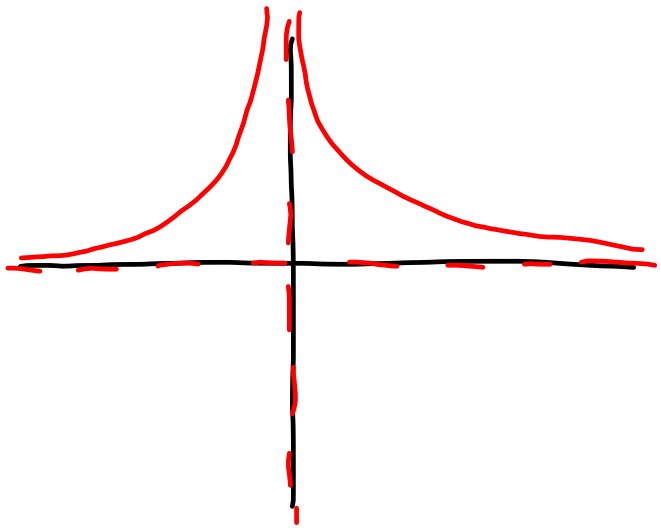
$\lim_{x\to 0^-} f(x) = -2$

$\lim_{x\to 3} \frac{|x-3|}{x-3}$ is undefined; $\lim_{x\to 3^+} f(x) = 1$, $\lim_{x\to 3^-} f(x) = -1$

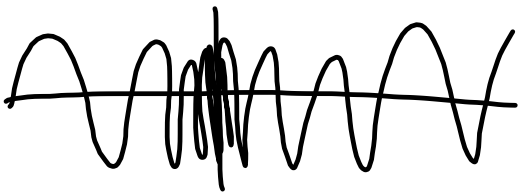
$$\frac{|x-3|}{x-3} = \begin{cases} \frac{x-3}{x-3}, & x-3>0 \\ \frac{-(x-3)}{x-3}, & x-3<0 \end{cases}$$

$$= \begin{cases} 1, & x>3 \\ -1, & x<3 \end{cases}$$

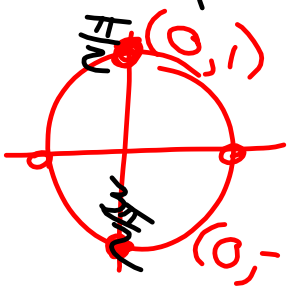
$\lim_{x\to 0} \frac{1}{x^4}$ is undefined $= \infty$



$\lim_{x \to 0} \sin \frac{1}{x}$



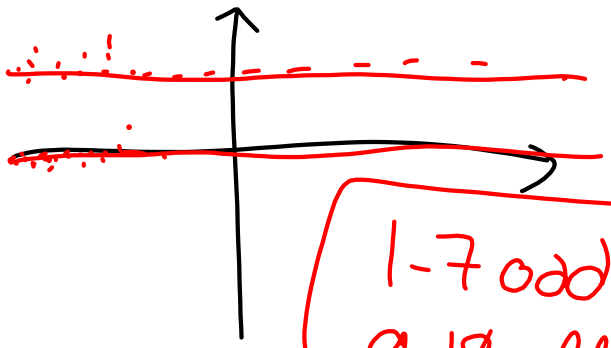
x	$\frac{2}{\pi}$	$\frac{2}{3\pi}$	$\frac{2}{5\pi}$	$\frac{2}{7\pi}$	$\frac{2}{9\pi}$	$\frac{2}{11\pi}$
$\sin \frac{1}{x}$	1	−1	1	−1	1	−1



limit is undefined
(b/c function continues to oscillate)

"Dirichlet Function"

$$f(x) = \begin{cases} 0, & \text{if } x \text{ is rational} \\ 1, & \text{if } x \text{ is irrational} \end{cases}$$



limit is undefined at all real numbers x

1-7 odd
9-18 all

#1-18