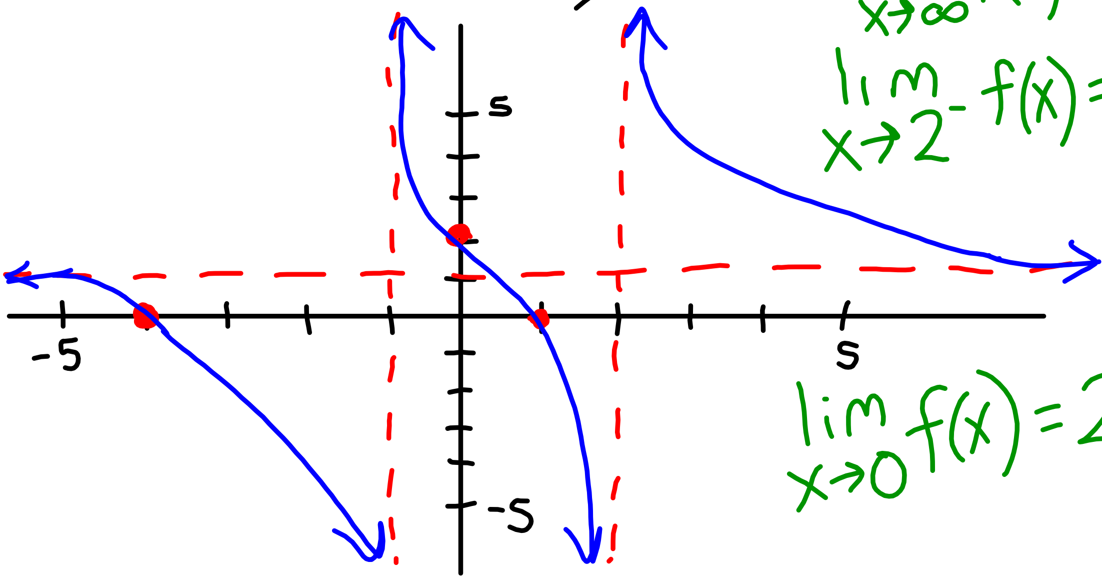
Graph the rational function.

$$f(x) = \frac{(x+4)(x-1)}{(x-2)(x+1)} \approx \frac{x^2}{x^2} = 1$$

$\lim_{x\to\infty} f(x) = 1$

$\lim_{x\to 2^-} f(x) = -\infty$



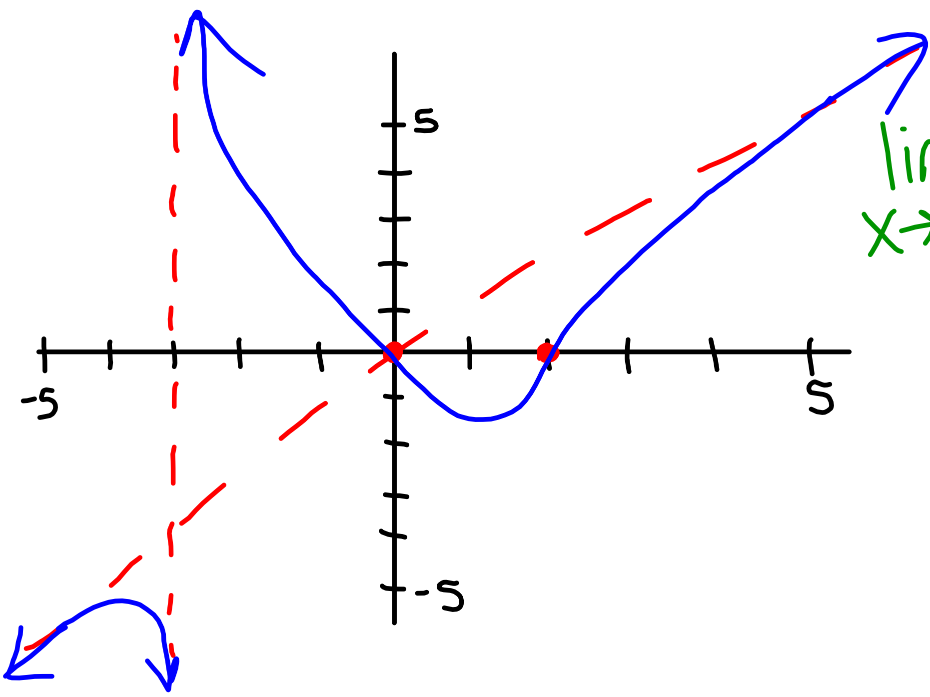
$\lim_{x\to 0} f(x) = 2$

$$f(x) = \frac{x(x-2)}{x+3} \approx \frac{x^2}{x} = x$$

$\lim_{x\to -\infty} f(x) = -\infty$

$\lim_{x\to -3^+} f(x) = \infty$

$\lim_{x\to 2} f(x) = 0$



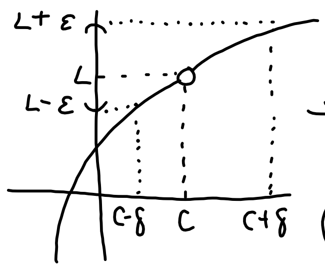
Informal Definition

If $f(x)$ becomes arbitrarily close to a single number L as x approaches c from either side, we say that the limit of $f(x)$ as x approaches c is L.

$$\lim_{x \to c} f(x) = L.$$

Building up to

ε-δ Definition of the Limit



"$f(x)$ becomes arbitrarily close to L"

→ $f(x)$ lies in the interval $(L-\varepsilon, L+\varepsilon)$ for some (really small) $\varepsilon > 0$.

ε = epsilon

δ = delta

$$|f(x) - L| < \varepsilon$$

"the distance between $f(x)$ and L is less than ε"

"x appoaches c" ⇒ there exists a positive # δ such that x is either in the interval $(c-\delta, c)$ or $(c, c+\delta)$.

$$0 < |x - c| < \delta.$$

$x \neq c$

ε-δ Def: Let f be a function defined on an open interval containing c (except possibly at c) and let L be a real number.
The statement $\lim_{x \to c} f(x) = L$ means that for each $\varepsilon > 0$, there exists a $\delta > 0$ such that if $0 < |x - c| < \delta$, then $|f(x) - L| < \varepsilon$.

$f(x) = 2x - 1$

Find $\lim_{x \to 4} f(x)$ and prove that is the limit using the ε-δ definition.

$\lim_{x \to 4} f(x) = 2(4) - 1 = 7 = L$, $\quad 4 = c$

Given $\varepsilon > 0$. We want to show that $\lim_{x \to 4} (2x - 1) = 7$, i.e., we want to find a $\delta > 0$ such that whenever $0 < |x - 4| < \delta$, we have $|f(x) - 7| < \varepsilon$.

$|f(x) - L| = |2x - 1 - 7| = |2x - 8| = 2|x - 4|$

We want $2|x - 4| < \varepsilon$

$|x - 4| < \varepsilon/2$

Take $\delta = \varepsilon/2$.

Then whenever $|x - c| = |x - 4| < \delta$,

$|f(x) - L| = |2x - 1 - 7| = 2|x - 4|$

$= 2|x - 4| < 2 \cdot \delta = 2 \cdot \varepsilon/2 = \varepsilon$.

[Translation: we found a δ so that $|x - c| < \delta$ implies $|f(x) - L| < \varepsilon$]

$f(x) = -5x+3$; find $\lim_{x \to 1} f(x)$ & find a δ.

$\lim_{x \to 1} f(x) = -5(1)+3 = -2$

$L = -2$; $C = 1$

$|f(x) - L| = |-5x+3-(-2)| = |-5x+5| =$

$= |-5(x-1)| = 5|x-1| < \varepsilon$

$|x-1| < \boxed{\varepsilon/5 = \delta}$

HW:

1.2 (page 56)

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& watch all the K.A. ε-δ videos!