$\epsilon - \delta$ Definition of the Limit:

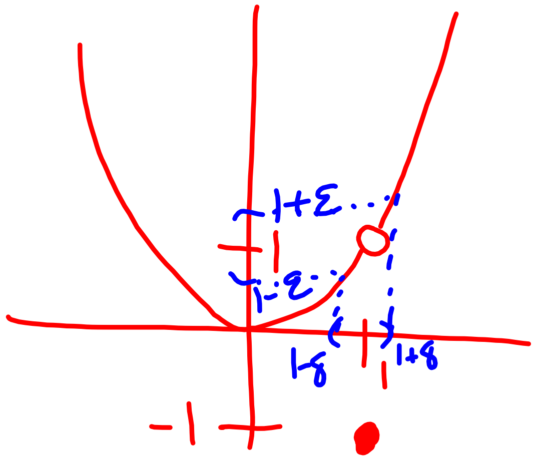
$\lim_{x \to c} f(x) = L$ *if given* $\epsilon > 0$,

there exists $\delta > 0$ *such that*

$|f(x) - L| < \epsilon$ *whenever* $|x - c| < \delta$.

$$f(x) = \begin{cases} x^2, & x \neq 1 \\ -1, & x = 1 \end{cases}$$

$$\lim_{x \to 1} f(x) = 1$$



Prove that the limit is L using the $\epsilon - \delta$ definition of the limit.

$L = -1$

$c = -3$

28. $\lim_{x \to -3} (2x + 5) = 2(-3) + 5 = -1$

$$|f(x) - L| = |2x + 5 - (-1)| = |2x + 6| =$$

$$= 2|x + 3| = 2|x - (-3)| < \varepsilon$$

$$|x - (-3)| < \boxed{\varepsilon/2 = \delta}$$

Proof:

Given $\varepsilon > 0$. Take $\delta = \varepsilon/2$. Then whenever $|x - (-3)| < \delta$, we have

$|2x + 5 - (-1)| = 2|x - (-3)| < 2\delta = 2 \cdot \frac{\varepsilon}{2} = \varepsilon$

i.e. $|f(x) - L| < \varepsilon$.

Find δ for $\epsilon = 0.01$

$L=2,\ c=4$

26. $\lim_{x \to 4}\left(4 - \frac{x}{2}\right) = 4 - \frac{4}{2} = 2$

$|f(x) - L| = \left|4 - \frac{x}{2} - 2\right| = \left|2 - \frac{x}{2}\right| =$

$= \left|-\frac{1}{2}\left(\frac{2}{-\frac{1}{2}} - \frac{\frac{x}{2}}{-\frac{1}{2}}\right)\right| = \left|-\frac{1}{2}(-4 + x)\right|$

$= \left|-\frac{1}{2}(x-4)\right| = \frac{1}{2}|x-4| < 0.01$

$\boxed{\delta = 0.02}$

whenever $|x-4| < 0.02$,

$|f(x) - L| = \left|4 - \frac{x}{2} - 2\right| = \frac{1}{2}|x-4| < \frac{1}{2}(0.02)$
$= 0.01$

Find δ for $\epsilon = 0.01$

$L=29$

$c=5$

26. $\lim_{x \to 5}(x^2 + 4) = 5^2 + 4 = 29$

$|f(x) - L| = |x^2 + 4 - 29| = |x^2 - 25| =$

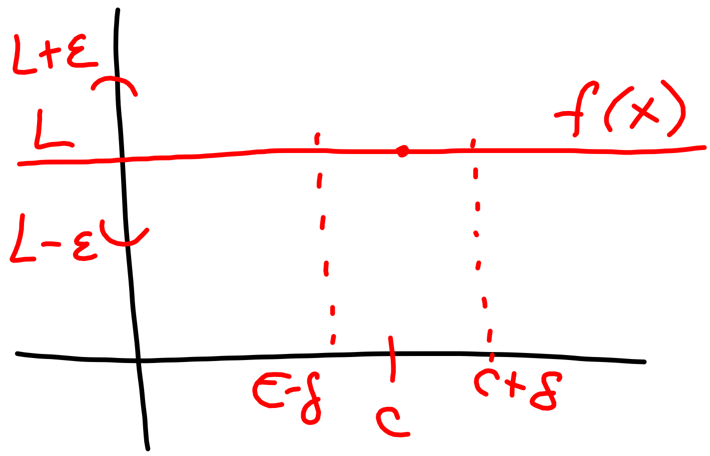
$|(x-5)(x+5)| \le |(x-5)(10+5)|$

$= 15|x-5| < 0.01$

$\delta = \frac{0.01}{15}$

Homework:

1.2 #23, 25; 30, 31



1.3 Evaluating Limits Analytically

If $\lim_{x \to c} f(x) = f(c)$,

we say that $f(x)$ is

continuous at c.

Basic Limits

$a, c \in \mathbb{R}$

$n \in \mathbb{N}$

$\lim\limits_{x \to c} a = a$

$\lim\limits_{x \to c} x = c$

$\lim\limits_{x \to c} x^n = c^n$

$\lim\limits_{x \to 5} (-3) = -3$

$\lim\limits_{x \to -\pi} x = -\pi$

$\lim\limits_{x \to -1} x^5 = (-1)^5 = -1$

Theorem 1.2 more properties of Limits

$b, c \in \mathbb{R}$, $n > 0$ an integer, f & g - functions

$\lim\limits_{x \to c} f(x) = L$; $\lim\limits_{x \to c} g(x) = K$

1. scalar multiple

$\lim\limits_{x \to c} [bf(x)] = bL$

2. sum or difference

$\lim\limits_{x \to c} [f(x) \pm g(x)] = L \pm K$

3. product

$\lim\limits_{x \to c} [f(x)g(x)] = LK$

4. quotient

$\lim\limits_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{L}{K}$, $K \neq 0$

5. power

$\lim\limits_{x \to c} [f(x)]^n = L^n$ (follows from #3)

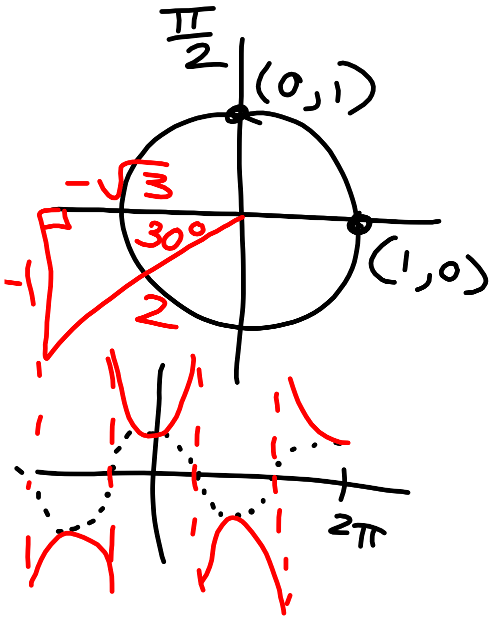
polynomials, rational functions, $\sqrt[n]{x}$, $f(g(x))$, sin, cos, etc.

1.3

12. $$\lim_{x \to 1} \left(3x^3 - 2x^2 + 4\right) = 3 - 2 + 4 = \boxed{5}$$

18. $$\lim_{x \to 3} \frac{\sqrt{x+1}}{x-4} = \frac{\sqrt{3+1}}{3-4} = \boxed{-2}$$

30. $$\lim_{x \to 1} \sin \frac{\pi x}{2} = \sin \frac{\pi}{2} = \boxed{1}$$



36. $$\lim_{x \to 7} \sec\left(\frac{\pi x}{6}\right)$$

$$= \sec \frac{7\pi}{6} = \boxed{\frac{-2}{\sqrt{3}}}$$

38. $\lim_{x\to c} f(x) = \frac{3}{2}$; $\lim_{x\to c} g(x) = \frac{1}{2}$

(a) $\lim_{x\to c} [4f(x)] = 4 \cdot \frac{3}{2} = \boxed{6}$

(b) $\lim_{x\to c} [f(x)+g(x)] = \frac{3}{2} + \frac{1}{2} = \frac{4}{2} = \boxed{2}$

(c) $\lim_{x\to c} [f(x)g(x)] = \frac{3}{2} \cdot \frac{1}{2} = \boxed{\frac{3}{4}}$

(d) $\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{3/2}{1/2} = \frac{3}{\cancel{2}} \cdot \frac{\cancel{2}}{1} = \boxed{3}$

Homework:

1.2 #23, 25; 30, 31

1.3
#11, 17, 27-35 odd, 39