25. $\varepsilon = 0.01$ ; find $\delta$

$$\lim_{x\to 2}(x^2-3) = 2^2-3 = 1 = L$$

$$|f(x)-L| = |x^2-3-1| = |x^2-4| = |(x+2)(x-2)|$$

$$< |(3+2)(x-2)| = \frac{5|x-2|}{5} < \frac{\varepsilon}{5} = \delta$$

$$\delta = \frac{0.01}{5} = 0.002$$

30. $\lim_{x\to 1}\left(\frac{2}{3}x+9\right) = \frac{29}{3}$

$$|f(x)-L| = \left|\frac{2}{3}x+9-\frac{29}{3}\right| = \left|\frac{2}{3}x-\frac{2}{3}\right|$$

$$= \frac{\frac{2}{3}|x-1|}{2/3} < \frac{\varepsilon}{2/3} = \delta \quad \boxed{\delta = \frac{3\varepsilon}{2}}$$

## 1.3 Evaluating Limits Analytically

42. $h(x) = \dfrac{x^2 - 3x}{x} = x - 3, \ x \neq 0$

(a) $\lim\limits_{x \to -2} h(x) = \dfrac{(-2)^2 - 3(-2)}{-2} = \dfrac{4+6}{-2} = \boxed{-5}$

(b) $\lim\limits_{x \to 0} h(x) = \dfrac{0^2 - 3(0)}{0} = \dfrac{0}{0}$ indeterminate form

$$\lim_{x \to 0} \frac{x^2 - 3x}{x} = \lim_{x \to 0} \frac{\cancel{x}(x-3)}{\cancel{x}} = 0 - 3 = \boxed{-3}$$

44. $\lim\limits_{x \to 1} \dfrac{x}{x^2 - x} = \dfrac{1}{1^2 - 1} = \dfrac{1}{0}$

$$= \lim_{x \to 1} \frac{x}{x(x-1)} = \lim_{x \to 1} \frac{1}{x-1} = \frac{1}{1-1} = \frac{1}{0}$$

$\boxed{\text{undefined}}$

$\boxed{\text{the limit does not exist}}$

48. $\lim_{x\to -1} \frac{x^3+1}{x+1} = \frac{(-1)^3+1}{-1+1} = \frac{0}{0}$

$a^3+b^3 = (a+b)(a^2-ab+b^2)$

$a^3-b^3 = (a-b)(a^2+ab+b^2)$

$\lim_{x\to -1} \frac{\cancel{(x+1)}(x^2-x+1)}{\cancel{x+1}} = (-1)^2-(-1)+1$

$= 1+1+1 = \boxed{3}$

54. $\lim_{x\to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x} = \frac{\sqrt{2}-\sqrt{2}}{0} = \frac{0}{0}$

$\lim_{x\to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x} \cdot \frac{\sqrt{2+x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{2}}$

$= \lim_{x\to 0} \frac{\cancel{2}+\cancel{x}-\cancel{2}}{\cancel{x}(\sqrt{2+x}+\sqrt{2})} = \frac{1}{\sqrt{2}+\sqrt{2}}$

$= \frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= \boxed{\frac{\sqrt{2}}{4}}$

$\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\frac{1}{2+\sqrt{3}} \cdot \frac{2-\sqrt{3}}{2-\sqrt{3}}$

58. $\lim_{x\to 0} \dfrac{\frac{1}{x+4} - \frac{1}{4}}{x}$ $\frac{4}{4}$ $\cdot\frac{x+4}{x+4}$ $= \lim_{x\to 0} \dfrac{\frac{4}{4(x+4)} - \frac{x+4}{4(x+4)}}{\frac{x}{1}}$

$= \lim_{x\to 0} \dfrac{\left(\frac{4-(x+4)}{4(x+4)}\right)}{\frac{x}{1}} = \lim_{x\to 0} \left(\frac{-\cancel{x}}{4(x+4)}\right)\cdot\frac{1}{\cancel{x}}$

$= \dfrac{-1}{4(4)} = \boxed{-\dfrac{1}{16}}$

62. $\lim_{\Delta x\to 0} \dfrac{(x+\Delta x)^3 - x^3}{\Delta x} = \lim_{h\to 0} \dfrac{(x+h)^3 - x^3}{h}$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ 1, 1 1, 1 3 3 1

$= \lim_{\Delta x\to 0} \dfrac{\cancel{x^3} + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - \cancel{x^3}}{\Delta x}$

$= \lim_{\Delta x\to 0} \dfrac{\cancel{\Delta x}\left(3x^2 + 3x\Delta x + (\Delta x)^2\right)}{\cancel{\Delta x}} = \boxed{3x^2}$

66. $\lim_{x \to 2} \frac{x^5 - 32}{x - 2}$

$(x^5 - 32) \div (x-2)$

$$\begin{array}{r|rrrrrr} 2 & 1 & 0 & 0 & 0 & 0 & -32 \\ & & 2 & 4 & 8 & 16 & 32 \\ \hline & 1x^4 & 2x^3 & 4x^2 & 8x & 16 & \boxed{0} \end{array}$$

$$= \lim_{x \to 2} \frac{\cancel{(x-2)}(x^4 + 2x^3 + 4x^2 + 8x + 16)}{\cancel{x-2}}$$

$$= 2^4 + 2^4 + 2^4 + 2^4 + 2^4$$

$$= \boxed{80}$$

<u>HW:</u> 1.3 # 41-44

51. $\lim_{x \to -3} \frac{x^2 + x - 6}{x^2 - 9}$

53. $\lim_{x \to 0} \frac{\sqrt{x+5} - \sqrt{5}}{x}$

55. $\lim_{x \to 4} \frac{\sqrt{x+5} - 3}{x - 4}$

57. $\lim_{x \to 0} \frac{\frac{1}{3+x} - \frac{1}{3}}{x}$

59. $\lim_{\Delta x \to 0} \frac{2(x+\Delta x) - 2x}{\Delta x}$

61. $\lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - 2(x+\Delta x) + 1 - (x^2 - 2x + 1)}{\Delta x}$

41-44 all
51-61 odd