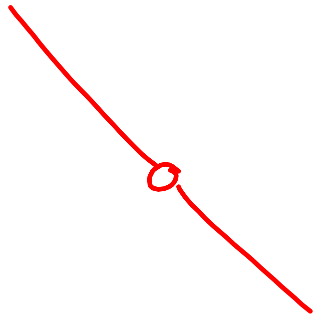
Review - Find the limits (if they exist):

1. $\lim_{x\to 2} f(x), where\ f(x) = \begin{cases} 4 - x, x \neq 2 \\ 0, \quad x = 2 \end{cases}$

2



2. $\lim_{x\to 1} f(x), where\ f(x) = \begin{cases} 4x - 7, x \neq 1 \\ 5, \quad x = 1 \end{cases}$

-3

3. $\lim_{x\to 0} \frac{|x|}{x} =$ does not exist!

is undefined

4. $\lim_{x\to 0} \frac{1 - \cos x}{x} =$

0

1.4

13. $\lim_{\Delta x \to 0^-} \dfrac{\frac{1}{x+\Delta x} - \frac{1}{x}}{\Delta x}$

$\lim_{h\to 0^-} \frac{x}{x} \dfrac{\frac{1}{x+h} - \frac{1}{x}}{h} \frac{x+h}{x+h} = \lim_{h\to 0^-} \frac{x-(x+h)}{x(x+h)} \cdot \frac{1}{h}$

$= \lim_{h\to 0^-} \frac{-h}{x^2+xh} \cdot \frac{1}{h} = \lim_{h\to 0^-} \frac{-1}{x^2+xh}$

$= -\frac{1}{x^2}$

1.4

39. $f(x) = \dfrac{x}{x^2+1}$

$x^2 \geq 0$

$x^2 + 1 \geq 1$

9. $\lim_{x \to -3^-} \dfrac{x}{\sqrt{x^2-9}}$

the limit does not exist

$\to -\infty$

defined

$x^2 - 9 > 0$

$(x-3)(x+3) > 0$

$+ \quad -3 \quad - \quad 3 \quad +$

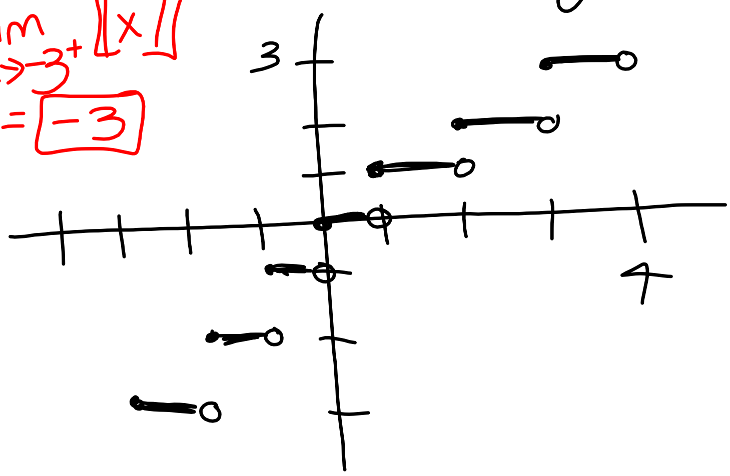
$(-\infty, -3) \cup (3, \infty)$

1.4 — (dis)continuity w/ Trig functions & [[x]] ; Intermediate Value Theorem

The Greatest Integer Function

$[[x]]$ = the greatest integer less than or equal to x

$\lim_{x \to -3^+} [[x]] = \boxed{-3}$



22. $\lim_{x\to 2^+} 2x - [\![x]\!]$

$= \lim_{x\to 2^+} 2x - \lim_{x\to 2^+} [\![x]\!]$

$= 4 - \lim_{x\to 2^+} [\![x]\!]$

$= 4 - 2 = \boxed{2}$

24. $\lim_{x\to 1} \left(1 - [\![-\frac{x}{2}]\!]\right)$

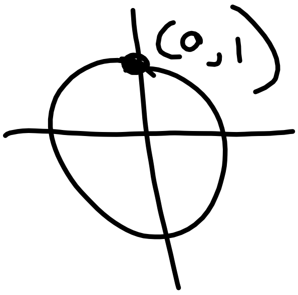
$= \lim_{x\to 1} 1 - \lim_{x\to 1} [\![-\frac{x}{2}]\!]$

$= 1 - \lim_{x\to 1} [\![-\frac{x}{2}]\!]$

$= 1 - (-1) = \boxed{2}$

20. $\lim_{x \to \frac{\pi}{2}} \sec x$

$= \lim_{x \to \frac{\pi}{2}} \frac{1}{\cos x} =$ undefined

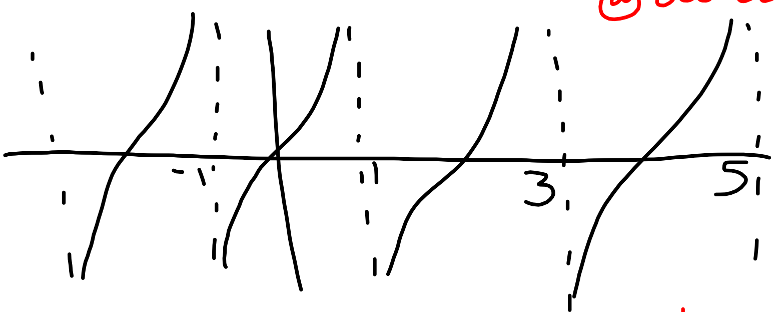


52. $f(x) = \tan \frac{\pi x}{2}$

discuss the (dis) continuity

period: $\frac{\pi}{\pi/2} = 2$

non-removable discontinuities @ all odd integers



f is continuous on all intervals of the form $(2n-1, 2n+1)$

62. $f(x)=\frac{1}{\sqrt{x}}, g(x)=x-1$

Discuss the continuity of $f(g(x))$.

$f(g(x))=\frac{1}{\sqrt{x-1}}$

$x-1>0$

$x>1$

f is continuous on $(1,\infty)$ (its domain)

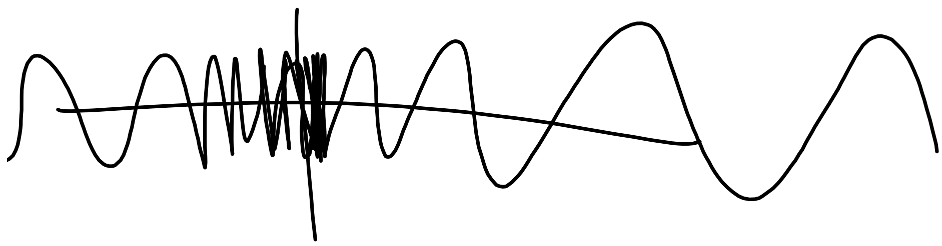
64. $f(x)=\sin x$; $g(x)=x^2$

discuss the continuity of $f(g(x))$

$f(g(x))=\sin(x^2)$

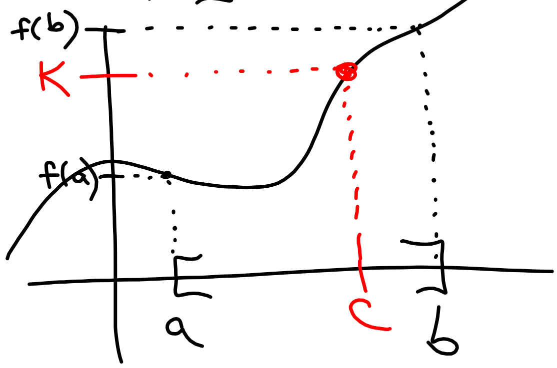
$g(f(x))=\sin^2 x$

continuous on $(-\infty,\infty)$



Intermediate Value Theorem

If f is continuous on the closed interval $[a,b]$ and K is any number between $f(a)$ and $f(b)$, then there is at least one number c in $[a,b]$ such that $f(c)=K$.



76. $f(x)=x^3+3x-2$, $[0,1]$

f is continuous on $[0,1]$ ✓

$f(0)=-2<0$

$f(1)=2>0$

$\Rightarrow$ IVT guarantees a zero in $[0,1]$

84. $f(x) = x^2 - 6x + 8$; $[0, 3]$; $f(c) = 0$

f is continuous on $[0,3]$ ✓

$f(0) = 8$

$f(3) = -1$

} IVT guarantees a zero

$x^2 - 6x + 8 = 0$

$(x-4)(x-2) = 0$

~~$x = 4$~~, $\boxed{x = 2}$

1.4

19, 21, 23, 51, 63, 69, 71, 83, 85