1.5

Infinite Limits

$$\lim_{x\to c} f(x) = \pm\infty$$

means the function increases or decreases without bound; i.e. the graph of the function approaches a vertical asymptote

Finding Vertical Asymptotes

x-values at which a function is undefined result in either holes in the graph or vertical asymptotes. Holes result when a function can be rewritten so that the factor which yields the discontinuity cancels. Factors that can't cancel yield vertical asymptotes.

Examples:

$f(x) = \frac{1}{x(x+3)}$ has vertical asymptotes at $x = 0$ and $x = 3$

$f(x) = \frac{(x+2)(x+3)}{x(x+3)}$ has a vertical asymptote at $x = 0$ and a hole at $x = -3$

Rules involving infinite limits

Let $\lim_{x\to c} f(x) = \infty \ and \ \lim_{x\to c} g(x) = L$

1. $\lim_{x\to c}[f(x) \pm g(x)] = \infty$

2. $\lim_{x\to c}[f(x)g(x)] = \begin{cases} \infty, & L > 0 \\ -\infty, & L < 0 \end{cases}$

3. $\lim_{x\to c} \frac{g(x)}{f(x)} = 0$

Find the vertical asymptotes (if any).

14. $f(x) = \dfrac{-4x}{x^2+4}$

$\boxed{\text{none}}$

$x^2 \geq 0$

$x^2+4 \geq 4$, i.e. denominator is never 0.

24. $h(x) = \dfrac{x^2-4}{x^3+2x^2+x+2} = \dfrac{\cancel{(x+2)}(x-2)}{\cancel{(x+2)}(x^2+1)}$

$x^2(x+2)+1(x+2)$

$ab+cb$

$b(a+c)$

hole @ -2

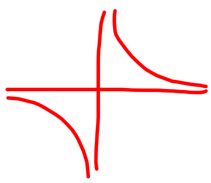
$\boxed{\text{none}}$

28. $g(\theta) = \dfrac{\tan\theta}{\theta} = \dfrac{\sin\theta}{\theta\cos\theta}$

$\boxed{x = \text{odd multiples of } \frac{\pi}{2}}$

$g(\theta)$ is undefined when $\theta = 0$, there is not a vertical asymptote there

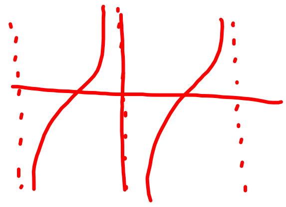
42. $\lim\limits_{x\to 0^-}\left(x^2 - \frac{1}{x}\right) = \lim\limits_{x\to 0^-}(x^2) - \lim\limits_{x\to 0^-}\left(\frac{1}{x}\right)$



$= 0 - (-\infty)$

$= \boxed{\infty}$

46. $\lim\limits_{x\to 0}\dfrac{x+2}{\cot x} = \dfrac{\lim\limits_{x\to 0}(x+2)}{\lim\limits_{x\to 0}\cot x} = \dfrac{2}{\pm\infty}$



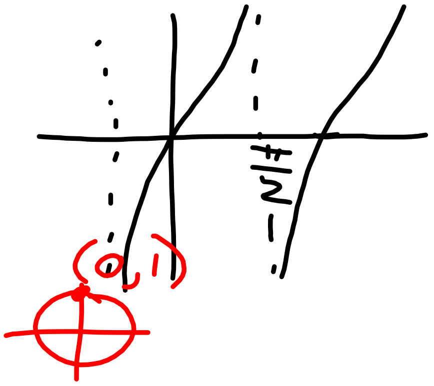
$= \boxed{0}$

Another way to look at this one:

$\lim\limits_{x\to 0}\dfrac{x+2}{\cot x} = \left[\lim\limits_{x\to 0}(x+2)\right]\left[\lim\limits_{x\to 0}\tan x\right] = 2\cdot 0$

$= 0$

48. $\lim_{x\to\frac{1}{2}} x^2\tan\pi x = \left(\lim_{x\to\frac{1}{2}} x^2\right)\left(\lim_{x\to\frac{1}{2}}\tan\pi x\right)$

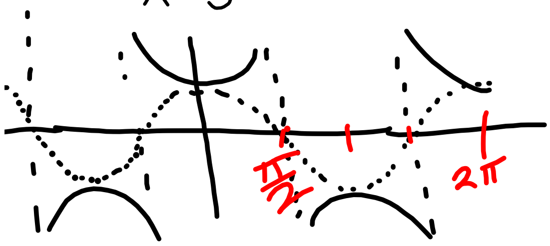


$= \frac{1}{4}\left(\lim_{x\to\frac{\pi}{2}}\tan x\right)$

$\pm\infty$

$\boxed{\text{undefined}}$

52. $\lim_{x\to3^+}\sec\frac{\pi x}{6} = \lim_{x\to\frac{\pi}{2}^+}\sec x = \boxed{-\infty}$



2.1 The Derivative & The Tangent Line Problem

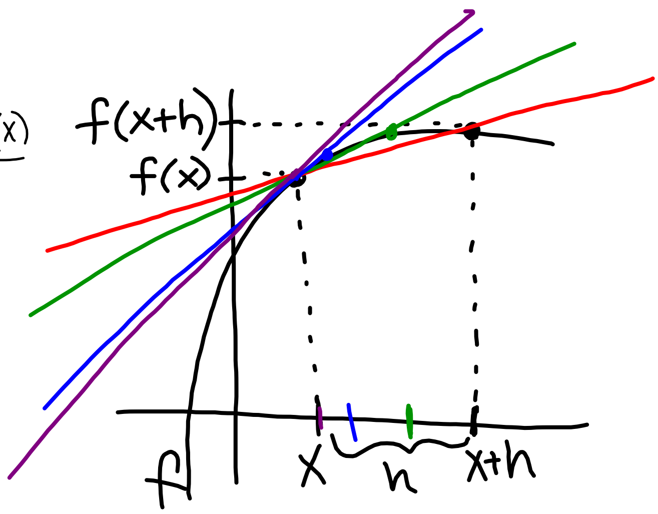
secant line
crosses through a function at two points

slope of the secant line:

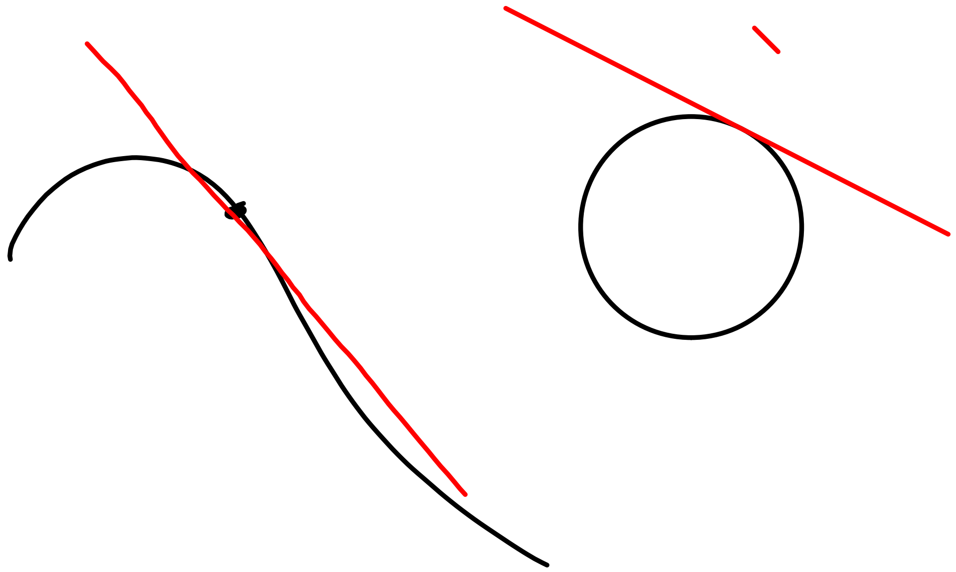
$$\frac{f(x+h)-f(x)}{x+h-x} = \frac{f(x+h)-f(x)}{h}$$

what happens as $h\to0$?

$$\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$$



As $h\to0$, the secant line approximates the tangent line, and the limit is the slope of the tangent line and we call it **<u>the derivative of f at x</u>**.



$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$\boldsymbol{f'(x)}$	"f prime of x"
$\boldsymbol{\frac{dy}{dx}}$	"derivative of y with respect to x"
$\boldsymbol{y'}$	"y prime"
$\boldsymbol{\frac{d}{dx}[f(x)]}$	"the derivative with respect to x of $f(x)$"
$\boldsymbol{D_x[y]}$	"the partial derivative with respect to x of y"

The Derivative

The slope of the tangent line to the graph of f at the point $(c, f(c))$ is given by:

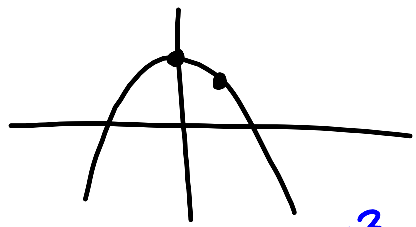
$$m = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x}$$

The derivative of f at x is given by

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

8. $g(x) = 5 - x^2$

find slope of tangent line at the points $\boxed{(2,1)}$ & $\boxed{(0,5)}$



$(c, g(c)) = (2, 1)$

$$m = \lim_{h \to 0} \frac{g(c+h) - g(c)}{h}$$

$$m\Big|_{(2,1)} = \lim_{h \to 0} \frac{5 - (2+h)^2 - 1}{h}$$

$$= \lim_{h \to 0} \frac{5 - (4 + 4h + h^2) - 1}{h} =$$

$$= \lim_{h \to 0} \frac{5 - 4 - 4h - h^2 - 1}{h}$$

$$= \lim_{h \to 0} \frac{-4h - h^2}{h} = \lim_{h \to 0} \frac{\cancel{h}(-4 - h)}{\cancel{h}} = \boxed{-4}$$

$$m\Big|_{(0,5)} = \lim_{h \to 0} \frac{5 - (0+h)^2 - 5}{h}$$

$$= \lim_{h \to 0} \frac{-h^2}{h} =$$

$$= \lim_{h \to 0} -h = \boxed{0}$$

20. $f(x) = x^3 + x^2$

find the derivative

Homework:

1.5 (infinite limits) - p.85 #1-51 odd

Ch 1 review pp. 88-89

2.1 (derivative definition) - p.101-102 #1-23odd

Old Test #1 on web
http://www.asms.net/brewer/calculus/diffcal-test1.pdf
(we will look at these on Friday)