2.1

20. $f(x)=x^3+x^2$

find the derivative

$$\begin{array}{c} 1 \\ 1 \quad 1 \\ 1 \quad 2 \quad 1 \\ 1 \quad 3 \quad 3 \quad 1 \\ 1 \quad 4 \quad 6 \quad 4 \quad 1 \end{array} \qquad \begin{array}{c} (a+b)^0 \\ (a+b)^1 \\ (a+b)^2 \\ (a+b)^3 \\ \end{array}$$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{(x+h)^3+(x+h)^2-(x^3+x^2)}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{x^3}+3x^2h+3xh^2+h^3+\cancel{x^2}+2xh+h^2-\cancel{x^3}-\cancel{x^2}}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{h}(3x^2+3xh+h^2+2x+h)}{\cancel{h}}=\boxed{3x^2+2x}$$

$f(x)=\frac{3}{x}$ Find $f'(x)$.

$$f'(x)=\lim_{h\to 0}\frac{\frac{3}{x+h}-\frac{3}{x}}{h}$$

$$=\lim_{h\to 0}\frac{3x-3(x+h)}{x(x+h)}\cdot\frac{1}{h}$$

$$=\lim_{h\to 0}\frac{3x-3x-3h}{(x^2+xh)h}=\lim_{h\to 0}\frac{-3\cancel{h}}{(x^2+xh)\cancel{h}}$$

$$=\boxed{\frac{-3}{x^2}}$$

1. Find the limit , then use the $\varepsilon - \delta$ definition to prove that the limit is L .

$\lim_{x\to 8}(3x-20) = 3(8)-20 = 24-20 = \boxed{4 = L}$

$|f(x)-L| = |3x-20-4| = |3x-24| = 3|x-8| < \varepsilon$

$|x-8| < \frac{\varepsilon}{3} = \delta$

Proof:

Given $\varepsilon > 0$, we want to find a $\delta > 0$ such that $|f(x)-L| < \varepsilon$ whenever $|x-c| < \delta$.

Take $\delta = \varepsilon/3$.

Then whenever $|x-c| < \delta$ i.e. $|x-8| < \varepsilon/3$, we have

$|f(x)-L| = |3x-20-4| = 3|x-8| < 3 \cdot \varepsilon/3 = \varepsilon$. □

2. Find the limit (if it exists).

$$\lim_{x\to -3}\frac{x^2+7x+12}{x^2-9} = \lim_{x\to -3}\frac{\cancel{(x+3)}(x+4)}{\cancel{(x+3)}(x-3)} = \frac{-3+4}{-3-3} = \boxed{-\frac{1}{6}}$$

3. Find the limit (if it exists).

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\quad, \ where\ f(x)=5x^2+3$$

Equivalent directions could have been "find the derivative" or "find f'(x)"

$$\lim_{h\to 0}\frac{5(x+h)^2+3-(5x^2+3)}{h}$$

$$=\lim_{h\to 0}\frac{5(x^2+2xh+h^2)+3-5x^2-3}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{5x^2}+10xh+5h^2\cancel{+3}-\cancel{5x^2}\cancel{-3}}{h}=\lim_{h\to 0}\frac{\cancel{h}(10x+5h)}{\cancel{h}}$$

$$=\boxed{10x}$$

4. Find the limit (if it exists).

$$\lim_{x\to 0}\frac{5\sin 2x}{3x}=\lim_{x\to 0}\underbrace{\frac{\sin 2x}{2x}}_{1}\cdot\underbrace{\left(\frac{5\cdot 2}{3}\right)}_{\frac{10}{3}}=\boxed{\frac{10}{3}}$$

5. Find the limit (if it exists).

$$\lim_{x\to -5} f(x), \quad f(x) = \begin{cases} -x^2+8, & x \le -5 \\ 2x+3, & x > -5 \end{cases}$$

$$-(-5)^2+8=-17$$

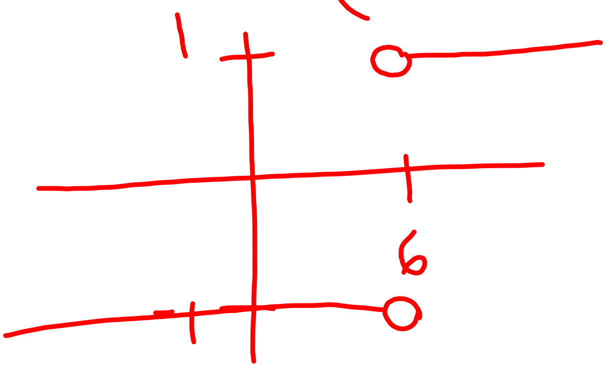
$$2(-5)+3=-7$$

$\Rightarrow$ $\boxed{\text{The limit does not exist}}$

6. Find the limit (if it exists). Show SOME sort of work, whether it's a graph or definition of absolute value of x-6 -- show me how you arrived at your answer.

$$\lim_{x\to 6^-} \frac{|x-6|}{x-6} = \boxed{-1}$$

$$\frac{|x-6|}{x-6} = \begin{cases} \frac{x-6}{x-6}=1, & x-6>0 \\ & x>6 \\ \frac{-(x-6)}{x-6}=-1, & x-6<0 \\ & x<6 \end{cases}$$



7. Use the Squeeze Theorem to find $\lim_{x \to 0} f(x)$. You <u>must</u> show use of the squeeze theorem.

$f(x) = 5x^2 \sin\frac{1}{x}$

$$-1 \le \sin\frac{1}{x} \le 1$$

$$-5x^2 \le 5x^2 \sin\frac{1}{x} \le 5x^2$$

$$\underbrace{\lim_{x \to 0}(-5x^2)}_{=0} \le \lim_{x \to 0}\left(5x^2 \sin\frac{1}{x}\right) \le \underbrace{\lim_{x \to 0}(5x^2)}_{=0}$$

$$\Rightarrow \lim_{x \to 0}\left(5x^2 \sin\frac{1}{x}\right) = \boxed{0}$$

8. Determine if the Intermediate Value Theorem guarantees a c in the interval $[-2, 3]$ such that $f(c) = -4$, and if so, find all such values of c.

$f(x) = x^2 - 7x + 2$

$f(-2) = (-2)^2 - 7(-2) + 2 = 4 + 14 + 2 = 20 > -4$

$f(3) = 3^2 - 7(3) + 2 = 9 - 21 + 2 = -10 < -4$

$\Rightarrow$ IVT guarantees a c in $[-2, 3]$

$f(c) = -4$ when $\cancel{c = 6}$ & $\boxed{c = 1}$

$c^2 - 7c + 2 = -4$

$c^2 - 7c + 6 = 0$

$(c-6)(c-1) = 0$

9. Discuss the continuity of the function (identify all discontinuities, if any, as removable or non-removable).

$$f(x) = \frac{x^2 - 7x + 10}{x^2 - 3x + 2} = \frac{(x-5)\cancel{(x-2)}}{(x-1)\cancel{(x-2)}}$$

$\Rightarrow$ removable discontinuity @ 2

nonremovable discontinuity @ 1

f is cts. on $(-\infty, 1) \cup (1,2) \cup (2,\infty)$

10. Find the limit (if it exists).

$$\lim_{x\to -2^+} \frac{\sqrt{x+11} - 3}{x^2 + 5x + 6} \cdot \frac{\sqrt{x+11} + 3}{\sqrt{x+11} + 3}$$

$$= \lim_{x\to -2^+} \frac{x+11-9}{(x+2)(x+3)(\sqrt{x+11}+3)} = \lim_{x\to -2^+} \frac{\cancel{x+2} \quad 1}{\cancel{(x+2)}(x+3)(\sqrt{x+11}+3)}$$

$$= \frac{1}{(-2+3)(\sqrt{-2+11}+3)} = \frac{1}{1(\sqrt{9}+3)} = \boxed{\frac{1}{6}}$$