2.1

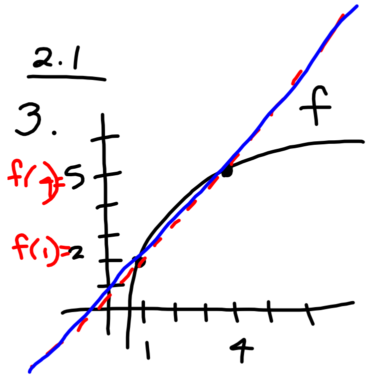
$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

11. $f(x)=3$

$$f'(x)=\lim_{h\to 0}\frac{3-3}{h}=\lim_{h\to 0}\frac{0}{h}=\lim_{h\to 0}0=0$$

2.1

3.



(a) $f(1)$ & $f(4)$ $=2$, $=5$

(b) $f(4)-f(1)=5-2=3$

(c) $y=\underbrace{\dfrac{f(4)-f(1)}{4-1}}_{\text{slope of line thru }(1,2)\,\&\,(4,5)\ =\frac{3}{3}=1}(x-1)+f(1)$ $=m$

$y-f(1)=m(x-1)$

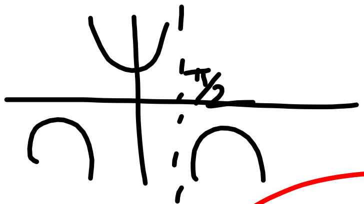
slope m & point $(1,f(1))$

$y-2=1(x-1)$ $\boxed{y=x+1}$

1.5

47. $\lim_{x\to\frac{1}{2}} x\sec\pi x=\left(\lim_{x\to\frac{1}{2}}x\right)\cdot\left(\lim_{x\to\frac{1}{2}}\sec\pi x\right)$

$= \frac{1}{2}$ $\quad$ $= +\infty$, $-\infty$



limit does not exist

$\lim_{x\to\frac{1}{2}^+}f(x)=-\infty$

$\quad -\quad =+\infty$

2.1

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

21. $f(x) = \frac{1}{x-1}$

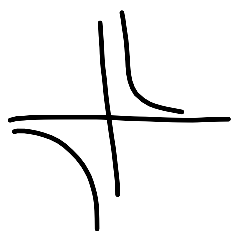
$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{x+h-1} - \frac{1}{x-1}}{\frac{h}{1}}$$

$$= \lim_{h \to 0} \frac{x-1-(x+h-1)}{(x+h-1)(x-1)\cdot h}$$

$$= \lim_{h \to 0} \frac{-\cancel{h}\,1}{(x+h-1)(x-1)\cdot \cancel{h}} = \boxed{\frac{-1}{(x-1)^2}}$$

1.5

41. $\lim_{x \to 0^-} \left(1 + \frac{1}{x}\right)$



$$= \lim_{x \to 0^-} (1) + \lim_{x \to 0^-} \frac{1}{x}$$

$$= 1 + (-\infty)$$

$$= \boxed{-\infty}$$

2.1 Differentiability & Continuity

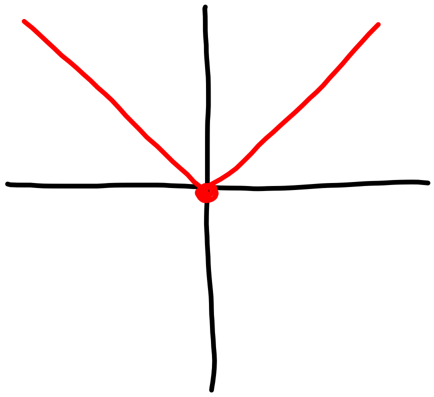
Alternative definition of the derivative at the point $(c, f(c))$:

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

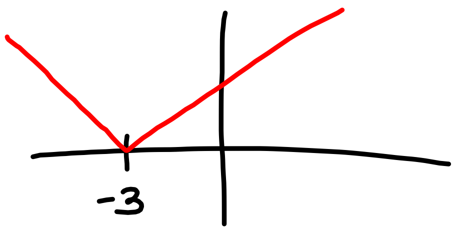
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

All differentiable functions are continuous, but not all continuous functions are differentiable.

e.g. $f(x) = |x|$



$f(x) = |x + 3|$



$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

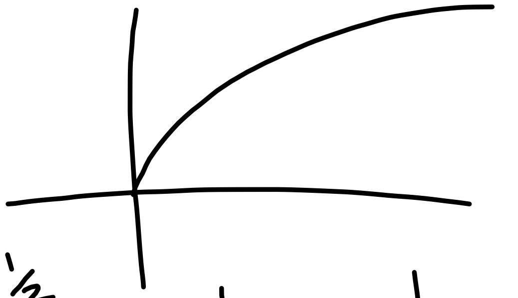
$$f'(-3) = \lim_{x \to -3} \frac{|x+3| - |-3+3|}{x - (-3)}$$

$$= \lim_{x \to -3} \frac{|x+3|}{x+3}$$

$$\left.\begin{aligned} \lim_{x \to -3^-} \frac{|x+3|}{x+3} &= -1 \\ \lim_{x \to -3^+} \frac{|x+3|}{x+3} &= 1 \end{aligned}\right\}$$

derivative does not exist

$f(x) = \sqrt{x}$



$$f'(0) = \lim_{x \to 0^+} \frac{\sqrt{x} - \sqrt{0}}{x - 0}$$

$$= \lim_{x \to 0^+} \frac{\sqrt{x}}{x} = \lim_{x \to 0^+} \frac{x^{1/2}}{x^1} = \lim_{x \to 0^+} \frac{1}{x^{1/2}}$$

$$= \lim_{x \to 0^+} \frac{1}{\sqrt{x}} = \infty$$

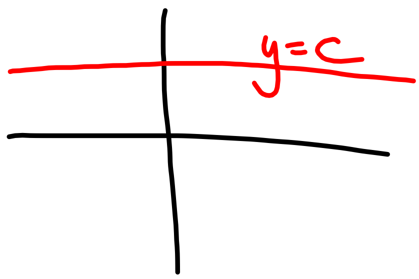
vertical tangent line

$\Rightarrow$ derivative does not exist

2.2 Basic Differentiation Rules

1. The derivative of a constant function is zero, i.e.,

for $c \in \mathbb{R}$, $\frac{d}{dx}[c] = 0$



Proof:

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{c - c}{h} = \lim_{h \to 0} \frac{0}{h} = \lim_{h \to 0} 0 = \boxed{0}$$

2. Power Rule for $n \epsilon \mathbb{Q}$, $\frac{d}{dx}[x^n] = nx^{n-1}$

Proof: $\lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$

Recall the binomial expansion:

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2}a^{n-2}b^2 + \cdots + \frac{n!}{k!\,(n-k)!}a^{n-k}b^k + \cdots + b^n$$

$$\lim_{h \to 0} \frac{\cancel{x^n} + nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 + \ldots \cancel{-x^n}}{h}$$

$$= \lim_{h \to 0} nx^{n-1} + \frac{n(n-1)}{2}x^{n-2} \cdot h + \ldots \to 0$$

$$= \boxed{nx^{n-1}}$$

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ for 29 $f(x) = x$

Special case: $\frac{d}{dx}[x] = 1$

Examples:

$$\frac{d}{dx}[x^7] = \lim_{h \to 0} \frac{(x+h)^7 - x^7}{h} = \ldots = 7x^6$$

$$\frac{d}{dx}[\pi^3] = 0$$

$$\frac{d}{dx}[2e] = 0$$

$$\frac{d}{dx}[\sqrt{x}] = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \frac{d}{dx}\left[x^{1/2}\right] = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-1/2} = \boxed{\frac{1}{2\sqrt{x}}}$$

$$\frac{d}{dx}\left[\frac{1}{x^3}\right] = \frac{d}{dx}\left[x^{-3}\right] = -3x^{-4} = \boxed{\frac{-3}{x^4}}$$