2.2

22. $y = 5 + \sin x$

$y' = 0 + \cos x = \boxed{\cos x}$

24. $y = \frac{5}{(2x)^3} + 2\cos x = \frac{5}{8}x^{-3} + 2\cos x$

$y' = \boxed{-\frac{15}{8}x^{-4} - 2\sin x}$

44. $h(x) = \frac{2x^3 - 3x + 1}{x} = \frac{2x^3}{x} - \frac{3x}{x} + \frac{1}{x} = 2x^2 - 3 + x^{-1}$

$h'(x) = \boxed{4x - x^{-2}}$

46. $y = 3x(6x - 5x^2) = 18x^2 - 15x^3$

$y' = \boxed{36x - 45x^2}$

52. $f(x) = \frac{2}{\sqrt[3]{x}} + 3\cos x = 2x^{-1/3} + 3\cos x$

$f'(x) = \boxed{-\frac{2}{3}x^{-4/3} - 3\sin x}$

Find the slope of the tangent line

$f(x) = 3x - \sin x \quad ; (\pi, 3\pi)$

$f'(x) = 3 - \cos x$

$m = f'(\pi) = 3 - \cos\pi = 3 - (-1) = \boxed{4}$

Find the equation of the tangent line.

$f(x) = 2x^3 + \sin x - 2x \; ; \; (0,0)$

$f'(x) = 6x^2 + \cos x - 2$

$y - y_1 = m(x - x_1)$

$m = f'(0) = 0 + 1 - 2 = -1$

$y - 0 = -1(x - 0)$

$\boxed{y = -x}$

2.2 cont.

$s(t)$ = position

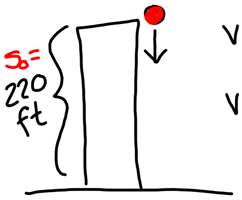
$v(t) = s'(t)$ = velocity

$a(t) = v'(t) = s''(t)$ = acceleration

average velocity: $\frac{\Delta s}{\Delta t}$ (slope of secant)

instantaneous velocity $= s'(t)$ (slope of tangent)

92.




initial velocity $v_0 = -22$ ft/s

$v(3) = ?$

$v(t) = ?$ after falling 108 ft

$s(t) = \frac{1}{2}gt^2 + v_0 t + s_0$

$g = -9.8 \; m/s^2$

$= -32 \; ft/s^2$

$s(t) = -16t^2 - 22t + 220$

$v(t) = -32t - 22$

$v(3) = -32(3) - 22 = -96 - 22 = \boxed{-118 \frac{ft}{s}}$

use $s(t)$ to find $t$ s.t. $s(t) = 108$

$108 = -16t^2 - 22t + 220$

$16t^2 + 22t - 112 = 0$

$8t^2 + 11t - 56 = 0$

$t = \frac{-11 \pm \sqrt{121 - 8(-56)}}{16} \approx 2.05s$

$v(2.05) =$

$-32(2.05) - 22$

$\approx \boxed{-87.6 \; ft/s}$

sphere volume: $V = \frac{4}{3}\pi r^3$

find the rate of change of volume w.r.t. radius when $r = 2$ cm.

$\frac{dV}{dr} = \frac{d}{dr}[V] = V' = \frac{d}{dr}\left[\frac{4}{3}\pi r^3\right]$

$V' = 4\pi r^2$ = surface area!

$V'(2) = 4\pi(2)^2 = \boxed{16\pi \; cm^2}$

# 2.3 Product & Quotient Rules

$$[fg]' = \frac{d}{dx}[f(x)g(x)] = f'(x)g(x)+f(x)g'(x)$$

$$\boxed{(fg)' = f'g + fg'}$$

$$\left[\frac{f}{g}\right]' = \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$$

"low dee high less high dee low, draw the line and square below"

$$\boxed{\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}}$$

2.3

6. $g(x) = \sqrt{x}\sin x = x^{1/2}\cdot\sin x$

$$g'(x) = (x^{1/2})'\cdot\sin x + x^{1/2}\cdot(\sin x)'$$

$$= \boxed{\frac{1}{2}x^{-1/2}\sin x + x^{1/2}\cos x}$$

12. $f(t) = \dfrac{\cos t}{t^3}$

$$f'(t) = \frac{t^3(\cos t)' - \cos t\,(t^3)'}{(t^3)^2}$$

$$= \boxed{\frac{-t^3\sin t - 3t^2\cos t}{t^6}}$$

$$= \boxed{-t^{-3}\sin t - 3t^{-4}\cos t}$$

Alt: $f(t) = (\cos t)\cdot(t^{-3})$

product rule:

$$f'(t) = (\cos t)'(t^{-3}) + (\cos t)(t^{-3})'$$

$$= \boxed{-t^{-3}\sin t - 3t^{-4}\cos t}$$

26. $f(x) = \dfrac{x^3+3x+2}{x^2-1}$

Note: as a product,

$f(x) = (x^3+3x+2)(x^2-1)^{-1}$

we don't know how to differentiate this yet so we have to use the quotient rule!

$$f'(x) = \frac{(x^2-1)(x^3+3x+2)' - (x^3+3x+2)(x^2-1)'}{(x^2-1)^2}$$

$$= \boxed{\frac{(x^2-1)(3x^2+3) - (x^3+3x+2)(2x)}{(x^2-1)^2}}$$

$$= \frac{3\cancel{x^4} + \cancel{3x^2} - \cancel{3x^2} - 3 - \cancel{2x^4} - 6x^2 - 4x}{x^4-2x^2+1}$$

$$= \boxed{\frac{x^4-6x^2-4x-3}{x^4-2x^2+1}}$$ ← simplified

## Derivatives of Trig Functions

1. $\frac{d}{dx}[\sin x] = \cos x$
2. $\frac{d}{dx}[\cos x] = -\sin x$
3. $\frac{d}{dx}[\tan x] = \sec^2 x$
4. $\frac{d}{dx}[\cot x] = -\csc^2 x$
5. $\frac{d}{dx}[\sec x] = \sec x \tan x$
6. $\frac{d}{dx}[\csc x] = -\csc x \cot x$

Power Rule:

$$\frac{d}{dx}[x^n] = nx^{n-1}$$

Constant Multiple Rule:

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$$

Sum & Difference:

$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$$

**Wednesday:**

**- product rule, quotient rule, chain rule**
**- quiz on basic derivative rules (those stated above)**

& statements of prod & quot. rules

Find the derivative by the limit process:
2.1 #21, 23

**Find the equation of the tangent line:**
**2.1 #29-32**

New!

Use the alternate form to find the derivative:
2.1 #61-69 odd

Describe the x-values where the function is differentiable (given a graph):
2.1 #71-79 odd

Find the derivative using the basic derivative rules we have learned so far:
2.2 #3-51 odd

**Use the derivative to solve rate of change word problems:**
**2.2 #91-94; 101,102**

New!

Work through intuitive exercises on **Khan Academy**:
- Slope of secant lines
- Tangent lope is limiting value of secant slope
- Derivative intuition
- Visualizing derivatives
- Graphs of functions and their derivatives
- The formal and alternate form of the derivative
- Derivatives 1
- Recognizing slopes of curves
- Power rule
- Special derivatives