# Quiz #2 Solutions

1. $\sec^2 x$
2. $f'(x)-g'(x)$
3. $f'(x)g(x)+f(x)g'(x)$
4. $\dfrac{f'(x)g(x)-f(x)g'(x)}{g^2(x)}$
5. $-\csc x \cot x$
6. $0$
7. $nx^{n-1}$
8. $e^x$
9. $\dfrac{1}{x\ln 3}$
10. $f'(g(x))\cdot g'(x)$
11. $10x-\cos x$
12. $[-\sin(4x^3-7)](12x^2)$
13. $3\sin x+3x\cos x$
14. $\dfrac{1}{(x+1)^2}$

13. $f(x)=(3x)\sin x$

$$f'(x)=(3x)'\cdot\sin x+(3x)(\sin x)'$$

$$=\boxed{3\sin x+3x\cos x}$$

14. $f(x)=\dfrac{x}{x+1}$

$$f'(x)=\frac{(x+1)(x)'-x(x+1)'}{(x+1)^2}=\frac{x+1-x}{(x+1)^2}$$

$$=\boxed{\frac{1}{(x+1)^2}}$$

$[x^n]' = nx^{n-1}$

$[cf(x)]' = cf'(x)$

$[f(x) \pm g(x)]' = f'(x) \pm g'(x)$

$[f(x)g(x)]' = f'g + fg'$

$\left[\frac{f(x)}{g(x)}\right]' = \frac{f'g - fg'}{g^2}$

$[f(g(x))]' = f'(g(x))\,g'(x)$

$[e^x]' = e^x$

$[a^x]' = a^x \ln a$

$[\ln x]' = \frac{1}{x}$

$[\log_a x]' = \frac{1}{x \ln a}$

$[\sin x]' = \cos x$

$[\cos x]' = -\sin x$

$[\tan x]' = \sec^2 x$

$[\cot x]' = -\csc^2 x$

$[\sec x]' = \sec x \tan x$

$[\csc x]' = -\csc x \cot x$

$[\arcsin x]' = \frac{1}{\sqrt{1-x^2}}$

$[\arctan x]' = \frac{1}{1+x^2}$

$[\text{arcsec}\, x]' = \frac{1}{|x|\sqrt{x^2-1}}$

$[\arccos x]' = -\frac{1}{\sqrt{1-x^2}}$

$[\text{arccot}\, x]' = -\frac{1}{1+x^2}$

$[\text{arccsc}\, x]' = -\frac{1}{|x|\sqrt{x^2-1}}$

$\arcsin x = \sin^{-1} x$

2.4 The Chain Rule, cont.

18. $f(x) = -3\sqrt[4]{2-9x} = -3(2-9x)^{1/4}$

$f'(x) = \frac{-3}{4}(2-9x)^{-3/4}(-9)$

32. $h(t) = \left(\frac{t^2}{t^3+2}\right)^2 = \frac{t^4}{t^6+4t^3+4}$

$h'(t) = \frac{(t^6+4t^3+4)(4t^3) - t^4(6t^5+12t^2)}{(t^6+4t^3+4)^2}$

50. $h(x) = \sec x^2$

$= \sec(x^2)$

$h'(x) = \sec(x^2)\tan(x^2) \cdot (2x)$

60. $g(t) = 5\cos^2 \pi t$ $= 5[\cos \pi t]^2$

$g'(t) = 10\cos \pi t \cdot (-\sin \pi t) \cdot \pi$

66. $y = \sin \sqrt[3]{x} + \sqrt[3]{\sin x}$ $= \sin(x^{1/3}) + (\sin x)^{1/3}$

$y' = \cos(x^{1/3}) \cdot \left(\frac{1}{3}x^{-2/3}\right) + \frac{1}{3}(\sin x)^{-2/3} \cdot \cos x$

5.4

46. $g(t) = e^{-3/t^2}$ $= e^{-3t^{-2}}$

$g'(t) = e^{-3t^{-2}} \cdot (6t^{-3})$

48. $y = \ln\left(\frac{1+e^x}{1-e^x}\right)$ $= \ln(1+e^x) - \ln(1-e^x)$

$y' = \frac{1}{1+e^x} \cdot e^x - \frac{1}{1-e^x} \cdot (-e^x)$

58. $y = \ln e^x$ $= x$ $\qquad y' = \frac{1}{e^x} \cdot e^x = 1$

$y' = 1$

5.5

46. $f(t) = \frac{3^{2t}}{t}$

$$f'(t) = \frac{t(3^{2t} \cdot \ln 3 \cdot 2) - 3^{2t} \cdot 1}{t^2}$$

54. $y = \log_{10} \frac{x^2 - 1}{x} = \log_{10}(x^2-1) - \log_{10} x$

$$y' = \frac{2x}{(x^2-1)\ln 10} - \frac{1}{x \ln 10}$$

5.8

$$[\operatorname{arcsec} x]' = [\sec^{-1}(x)]' = \frac{1}{|x|\sqrt{x^2-1}}$$

44. $f(x) = \operatorname{arcsec} 2x$

$$f'(x) = \frac{1}{|2x|\sqrt{(2x)^2-1}} \cdot 2 = \frac{1}{|x|\sqrt{4x^2-1}}$$

48. $h(x) = x^2 \arctan x$

$$h'(x) = 2x \arctan x + x^2 \cdot \frac{1}{1+x^2}$$

52. $y = \ln(t^2+4) - \frac{1}{2}\arctan\frac{t}{2}$

$$y' = \frac{2t}{t^2+4} - \frac{1}{2} \cdot \left(\frac{1}{1+(t/2)^2}\right) \cdot \frac{1}{2}$$

56. $y = x \arctan 2x - \frac{1}{4}\ln(1+4x^2)$

$y' = \arctan 2x + x \cdot \frac{2}{1+(2x)^2} - \frac{1}{4} \cdot \frac{8x}{1+4x^2}$

$= \arctan 2x + \frac{2x}{1+4x^2} - \frac{2x}{1+4x^2} = \boxed{\arctan 2x}$

5.4 – Find the second derivative

80. $f(x) = \frac{1}{x-2} = (x-2)^{-1}$

$f'(x) = -(x-2)^{-2}$

$f''(x) = \boxed{2(x-2)^{-3}}$

82. $f(x) = \sec^2 \pi x = [\sec \pi x]^2$

$f'(x) = 2\sec \pi x \cdot \sec \pi x \tan \pi x \cdot \pi$

$= 2\pi [\sec^2 \pi x] \tan \pi x$

$f''(x) = 2\pi \sec^2 \pi x (\tan \pi x)' + 2\pi (\sec^2 \pi x)' \tan \pi x$

$= 2\pi \sec^2 \pi x \cdot \sec^2 \pi x \cdot \pi + 2\pi (2\pi \sec^2 \pi x \tan \pi x) \tan \pi x$

$= 2\pi^2 \sec^4 \pi x + 4\pi^2 \sec^2 \pi x \tan^2 \pi x$

5.4 Find the equation of the tangent line to the graph of $f$ at the indicated point.

78. $f(x) = \tan^2 x \;;\; \left(\frac{\pi}{4}, 1\right)$ $[\tan x]^2$

$f'(x) = 2\tan x \cdot \sec^2 x$

$m = f'\left(\frac{\pi}{4}\right) = 2\cdot\tan\left(\frac{\pi}{4}\right)\cdot\left(\sec\frac{\pi}{4}\right)^2$

$= 2\cdot 1\cdot(\sqrt{2})^2$

$= 4$

$y - 1 = 4\left(x - \frac{\pi}{4}\right)$

$\boxed{y = 4x - \pi + 1}$

5.1

$\log a^b = b\cdot\log a$

58. $f(x) = \ln\sqrt[3]{\dfrac{x-1}{x+1}}$

$= \ln\left(\frac{x-1}{x+1}\right)^{1/3} = \frac{1}{3}\ln\left(\frac{x-1}{x+1}\right) =$

$= \frac{1}{3}\left[\ln(x-1) - \ln(x+1)\right]$

$= \frac{1}{3}\ln(x-1) - \frac{1}{3}\ln(x+1)$

$f'(x) = \boxed{\frac{1}{3(x-1)} - \frac{1}{3(x+1)}}$

5.8-ish

$f(x) = \arcsin(3x)$

$f'(x) = \dfrac{3}{\sqrt{1-(3x)^2}}$

$f(x) = \arctan(\ln(2x))$

$f'(x) = \dfrac{1}{1+(\ln 2x)^2} \cdot \dfrac{1}{2x} \cdot 2$

$f(x) = \cot\left(5^{\arcsin(4x^3)}\right)$

$f'(x) = -\csc^2\left(5^{\arcsin(4x^3)}\right) \cdot 5^{\arcsin(4x^3)} \cdot \ln 5 \cdot \dfrac{1}{\sqrt{1-(4x^3)^2}} \cdot 12x^2$

$\left(a^{f(x)}\right)' = a^{f(x)} \cdot \ln a \cdot f'(x)$

## Test #2 - Thursday?

2.1 derivave definion

#21, 23, 29-32all, 61-79odd

2.2 basic differenaon rules

#3-51odd, 91-94all, 101, 102

2.3 product and quoent rules

#1-53odd,63-69odd,75-81all,83-91odd,109-115all

2.4 chain rule #7-81odd

5.1 logarithmic funcons #45-61, 71

5.4 exponenal funcons #39-57

5.5 log and exp funcons with other bases #41-55

5.8 inverse trig funcons #41-59