$$[f(x)]'' = [f'(x)]'$$

$$[\sin(x)]'' = (\cos x)' = -\sin x$$

$$[x^5]'' = (5x^4)' = 20x^3$$

$$\frac{df}{dx}, \frac{d^2f}{dx^2}, \frac{d^3f}{dx^3}, \frac{d^4f}{dx^4}, \dots, \frac{d^nf}{dx^n}$$

$$f'(x), f''(x), f^{(3)}(x), f^{(4)}(x), \dots, f^{(n)}(x)$$

$$f(x) = 3x^4 - 2x^3 + 5x^2 - 3x + 2$$

$$f'(x) = 12x^3 - 6x^2 + 10x - 3$$

$$f''(x) = 36x^2 - 12x + 10$$

$$f^{(3)}(x) = 72x - 12$$

$$f^{(4)}(x) = 72$$

$$f^{(5)}(x) = 0$$

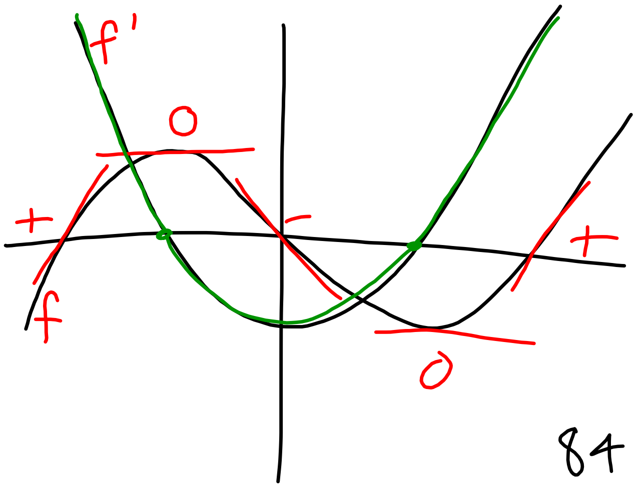
$$f^{(6)}(x) = 0$$

$$f^{(7)}(x) = 0$$

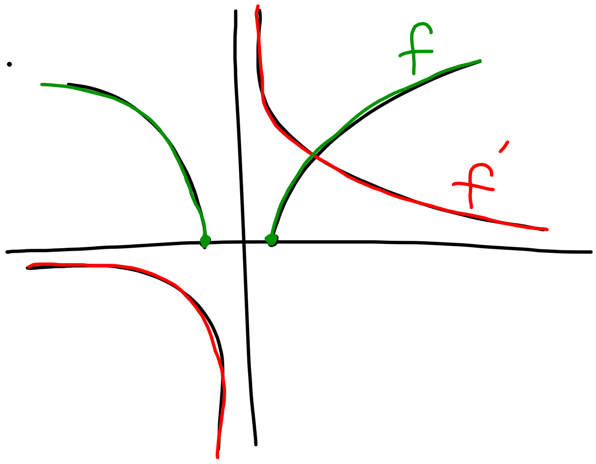
$$f^{(n)}(x) = 0, n \geq 5$$

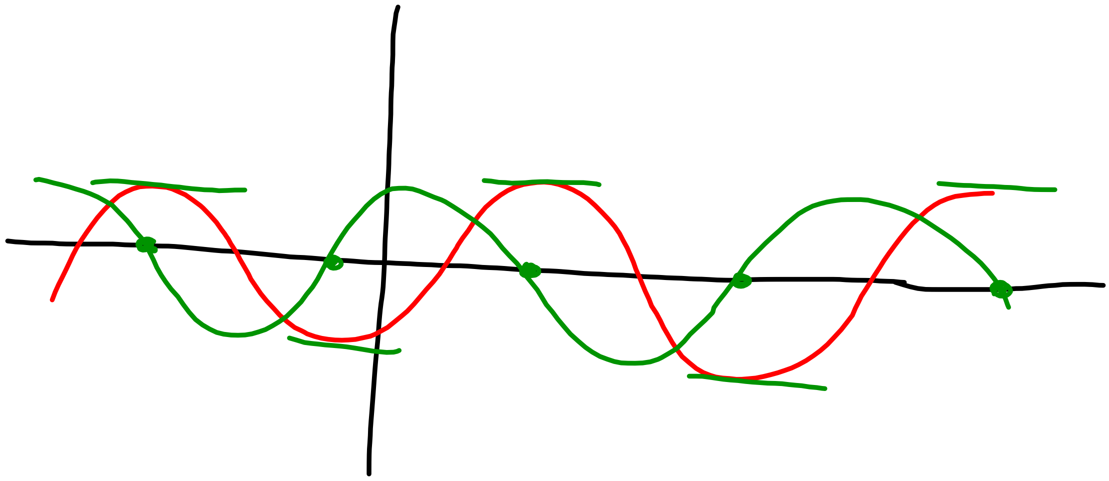
$f^{(23)}(x)$ of $f(x) = 7x^{20} - 5x^4$

$f^{(23)}(x) = 0$



84.





What happens if . . .

$$x^2y + y^2x = -2$$

how to find y'?

$$x^2 + y^2 = 1$$

$$y = \pm\sqrt{1-x^2}$$

2.5 Implicit Differentiation

$$y = f(x)$$

y is a function of x

$$\frac{d}{dx}[x] = 1 \quad ; \quad \frac{d}{dx}[y] = y'$$

6. $x^2y + y^2x = -2$

$$\frac{d}{dx}\left[x^2y + y^2x\right] = \frac{d}{dx}[-2]$$

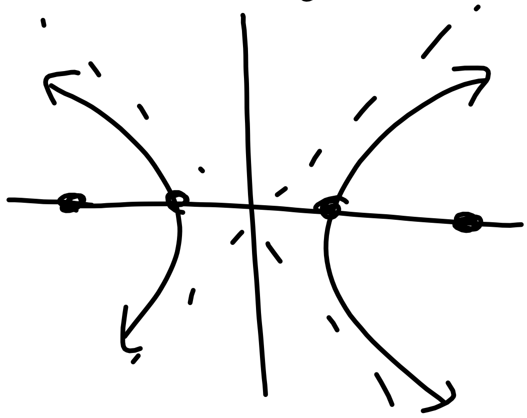
$$x^2(y)' + (x^2)'y + y^2(x)' + (y^2)'x = 0$$

$$\underline{x^2y'} + 2xy + y^2 + \underline{2yy'x} = 0$$

$$y'(x^2 + 2xy) = -y^2 - 2xy$$

$$\boxed{y' = \frac{-y^2 - 2xy}{x^2 + 2xy}}$$

2. $x^2-y^2=16$



$$\frac{d}{dx}(x^2-y^2)=\frac{d}{dx}(16)$$

$$2x-2y\cdot y'=0$$

$$-2yy'=-2x$$

$$y'=\frac{\cancel{-2}x}{\cancel{-2}y}$$

$$\boxed{y'=\frac{x}{y}}$$

8. $\sqrt{xy}=x-2y$ $(xy)^{1/2}=x-2y$

$$\frac{1}{2}(xy)^{-1/2}(xy)'=x'-(2y)'$$

$$\frac{1}{2}(xy)^{-1/2}(xy'+y)=1-2y'$$

$$\frac{xy'}{2\sqrt{xy}}+\frac{y}{2\sqrt{xy}}=1-2y'$$

$$y'\left(\frac{x}{2\sqrt{xy}}+2\right)=1-\frac{y}{2\sqrt{xy}}$$

$$\boxed{y'=\frac{1-\frac{y}{2\sqrt{xy}}}{\frac{x}{2\sqrt{xy}}+2}}$$

10. $2\sin x\cos y = 1$

$(2\sin x\cos y)' = 1'$

$2\sin x(\cos y)' + (2\sin x)'\cos y = 0$

$2\sin x(-\sin y\cdot y') + 2\cos x\cos y = 0$

$-2\sin x\sin y\cdot y' = -2\cos x\cos y$

$y' = \dfrac{-2\cos x\cos y}{-2\sin x\sin y} = \boxed{\cot x\cot y}$

12. $(\sin\pi x + \cos\pi y)^2 = 2$

$2(\sin\pi x+\cos\pi y)(\sin\pi x+\cos\pi y)' = 0$

$2(\sin\pi x+\cos\pi y)(\pi\cos\pi x - \pi y'\sin\pi y) = 0$

$2\pi\sin\pi x\cos\pi x - 2\pi y'\sin\pi x\sin\pi y + 2\pi\cos\pi x\cos\pi y$
$- 2\pi y'\sin\pi y\cos\pi y = 0$

$2\pi\sin\pi x\cos\pi x + 2\pi\cos\pi x\cos\pi y = y'(2\pi\sin\pi x\sin\pi y + 2\pi\sin\pi y\cos\pi y)$

$y' = \dfrac{\cancel{2\pi}\sin\pi x\cos\pi x + \cancel{2\pi}\cos\pi x\cos\pi y}{\cancel{2\pi}\sin\pi x\sin\pi y + \cancel{2\pi}\sin\pi y\cos\pi y}$

$= \dfrac{\cos\pi x\,\cancel{(\sin\pi x+\cos\pi y)}}{\sin\pi y\,\cancel{(\sin\pi x+\cos\pi y)}}$

$= \boxed{\dfrac{\cos\pi x}{\sin\pi y}}$

16. $x = \sec\frac{1}{y}$

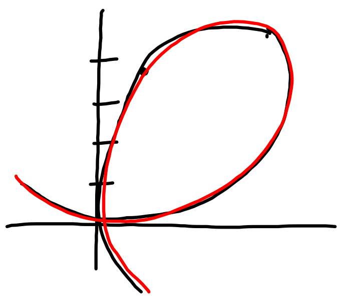
$x' = [\sec(y^{-1})]'$

$1 = \sec\frac{1}{y}\tan\frac{1}{y}\cdot(-y^{-2}\cdot y')$

$1 = \sec\frac{1}{y}\tan\frac{1}{y}\left(-\frac{y'}{y^2}\right)$

$y' = \boxed{-y^2\cos\frac{1}{y}\cot\frac{1}{y}}$

32. Folium of Descartes



$x^3+y^3-6xy=0$

find the slope of the tangent line @ $\left(\frac{4}{3},\frac{8}{3}\right)$

$3x^2+3y^2y'-6xy'-6y=0$

$y'(3y^2-6x) = 6y-3x^2$

$y' = \frac{6y-3x^2}{3y^2-6x} = \frac{2y-x^2}{y^2-2x}$

$m = y'\Big|_{\left(\frac{4}{3},\frac{8}{3}\right)} = \frac{2\left(\frac{8}{3}\right)-\left(\frac{4}{3}\right)^2}{\left(\frac{8}{3}\right)^2-2\left(\frac{4}{3}\right)} =$

40. Find y'' in terms of x & y.

$y^2 = 4x$

2.5 # 1-39odd; 43, 47