Homework questions?

**Quiz** #3- ~~Tues~~ Wed Oct ~~1~~ 2
(implicit differentiation, rates of change)
Test #3 - Fri Oct 11?

2.5 #15

$y = \sin(xy)$

$y' = \cos(xy) \cdot [xy' + y]$

$y' = xy'\cos xy + y\cos xy$

$y'(1 - x\cos xy) = y\cos xy$

$$y' = \frac{y\cos xy}{1 - x\cos xy}$$

2.5 # 43

$x^2 + y^2 = 25$ ; $(4,3), (-3,4)$

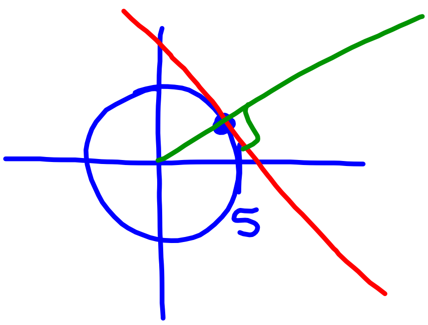
$2x + 2yy' = 0$

$$y' = \frac{-2x}{2y} = \frac{-x}{y}$$

tangent line @ $(4,3)$

$m = -\frac{4}{3}$

$y - 3 = -\frac{4}{3}(x - 4)$

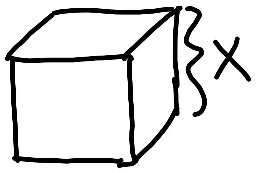



normal line @ $(4,3)$

$m = \frac{3}{4}$

$y - 3 = \frac{3}{4}(x - 4)$

2.6 #21




$$\frac{dx}{dt} = 3\,cm/s$$

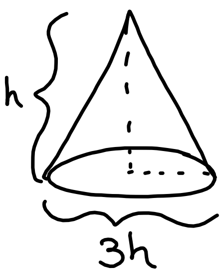
$$\frac{dA}{dt} = ? \text{ when } x = 1\text{ cm}$$

$$A = 6x^2$$

$$\frac{dA}{dt} = 12x \cdot \frac{dx}{dt} = 12(1)(3) = \boxed{36\ cm^2/s}$$

2.6 #23

$$r = \frac{3}{2}h$$




$$\frac{dV}{dt} = 10\ ft^3/min$$

$$\frac{dh}{dt} = ? \text{ when } h = 15\text{ ft}$$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{3}{2}h\right)^2 h = \frac{3\pi}{4}h^3$$

$$\frac{dV}{dt} = \frac{9\pi}{4}h^2 \cdot \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{\frac{dV}{dt}}{\frac{9\pi}{4}h^2} = \frac{\overset{2}{\cancel{10}} \cdot 4}{9\pi \cdot \underset{3\cdot 5\cdot 3\cdot \cancel{5}}{15^2}} = \boxed{\frac{8}{405\pi}\ ft/min}$$

## 3.1 Extrema on an Interval

22. $f(x) = x^3 - 12x$ , $[0, 4]$

$f'(x) = 3x^2 - 12$

$3x^2 - 12 = 0$

$3x^2 = 12$

$x^2 = 4$

$x = \pm 2$

critical # in $[0,4]$: 2

$f(0) = 0$

$f(2) = -16$

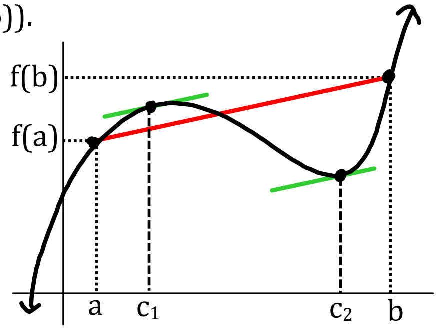
$f(4) = 16$

Abs. max. is 16 @ $x = 4$

Abs. min is $-16$ @ $x = 2$

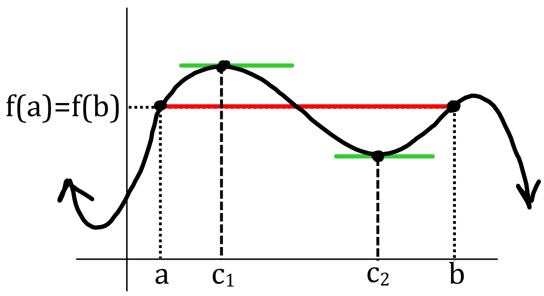
## 3.2 Rolle's Theorem & The Mean Value Theorem

The Mean Value Thereom (MVT) states: If f is continuous on [a,b] and differentiable on (a,b), then there exists at least one c in (a,b) such that the slope of the tangent line at c is equal to the slope of the secant line through (a, f(a)) and (b, f(b)).




$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

Rolle's Theorem is a special case of the MVT where f(a)=f(b), (and hence involving horizontal secant/tangent lines.




$$0 = f'(c)$$

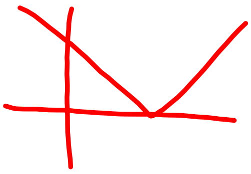
Note that neither the Mean Value Theorem nor Rolle's Theorem apply to the following functions on the given intervals:

$$f(x) = \frac{x+5}{x-2}\ , \qquad [1,3]$$

$f$ is not continuous on [1,3].

$$g(x) = |x-2|\ , \qquad [1,3]$$

$g$ is continuous on [1,3], but not differentiable on (1,3).



Can Rolle's Theorem be applied?
If so, ind all guaranteed values of c in (a,b).

8. $f(x) = x^2 - 5x + 4\ ,\ [1,4]$

f cts. on [1,4]? yes
f diff. on (1,4)? yes
$f(1) = 0$
$f(4) = 0$ =

Rolle's Thm Applies

$f'(x) = 2x - 5$
$2x - 5 = 0$
$2x = 5$
$x = 5/2 \in [1,4]$

$\boxed{c = 5/2}$

Can the Mean Value Theorem be applied?
If so, ind all guaranteed values of c in (a,b).

34. $f(x) = \frac{x+1}{x}$, $\left[\frac{1}{2}, 2\right]$

<u>Steps to solve MVT problems</u>:

1. Is f continuous on [a,b]? yes
2. Is f differentiable on (a,b)? yes } MVT applies
3. Find (f(b)-f(a))/(b-a) $= -1$
4. Find f'(x) $= -1/x^2$
5. Set #3&4 equal, solve for x
6. Solution is the values of x from #5 that lie in (a,b) $\rightarrow \boxed{c = 1}$

$$\frac{f(2) - f(\frac{1}{2})}{2 - \frac{1}{2}} = \frac{\frac{2+1}{2} - \frac{\frac{1}{2}+1}{\frac{1}{2}}}{\frac{4}{2} - \frac{1}{2}} = \frac{\frac{3}{2} - 3}{\frac{3}{2}} = -1$$

$$f'(x) = \frac{x(1) - (x+1)}{x^2} = \frac{-1}{x^2}$$

$$\frac{-1}{x^2} = -1$$

$$-1 = -x^2$$

$$1 = x^2$$

$$\pm 1 = x$$

38. $f(x) = 2\sin x + \sin 2x$, $[0, \pi]$

f cts on $[0,\pi]$? yes
f diff on $(0,\pi)$? yes } MVT applies

$$\frac{f(\pi) - f(0)}{\pi - 0} = \frac{2\sin\pi + \sin 2\pi - (2\sin 0 + \sin 2(0))}{\pi} = 0$$

$$f'(x) = 2\cos x + 2\cos 2x$$

$$2\cos x + 2\cos 2x = 0$$

$$\cos x + \cos 2x = 0$$

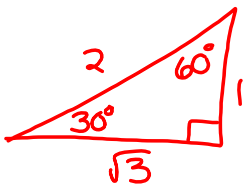
$$\cos x + (2\cos^2 x - 1) = 0$$

$$2\cos^2 x + \cos x - 1 = 0$$

$$(2\cos x - 1)(\cos x + 1) = 0$$

$\cos x = 1/2$ $\quad$ $\cos x = -1$

$\boxed{x = \pi/3}$ $\quad$ $x = \pi \notin (0,\pi)$



32. $f(x) = x(x^2 - x - 2)$ $[-1, 1]$

$f$ cts on $[-1,1]$? yes
$f$ diff. on $(-1,1)$? yes
} MVT applies

$$\frac{f(1) - f(-1)}{1-(-1)} = -1$$

$f(x) = x^3 - x^2 - 2x$

$f'(x) = 3x^2 - 2x - 2$

$3x^2 - 2x - 2 = -1$

$3x^2 - 2x - 1 = 0$

$3x^2 - 3x + x - 1 = 0$

$3x(x-1) + 1(x-1) = 0$

$(x-1)(3x+1) = 0$

$x = 1$, $x = -1/3$

3.3-3.4 Increasing, Decreasing, Concavity, and the 1st and 2nd Derivative Tests

## What do $f'$ and $f''$ tell us about $f$?

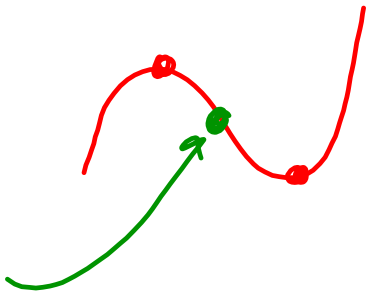
Recall that $f'$ is the rate of change or slope of $f$,
$f''$ is the slope or rate of change of $f'$.

| $f'$ | $f$ |
|---|---|
| + | ↗ increasing |
| − | ↘ decreasing |

| $f''$ | $f'$ | $f$ |
|---|---|---|
| + | ↗ increasing | concave up |
| − | ↘ decreasing | concave down |

*f'(x)=0* when f has a relative maximum or minimum.
These x-values (and those where f'(x) is undeined) are called <u>critical numbers</u>.

*f''(x)=0* when f changes concavity.
The points where concavity changes are called <u>inlection points</u> .



To solve problems involving concavity, increasing/decreasing, etc., we should recall how to solve polynomial inequalities.

$$\frac{(x+2)(x-3)}{x+4} \geq 0$$

Homework since Test #2 (Material for Test #3)

2.5 # 1-39 odd; 43, 47 - Implicit Differentiation
2.6 # 15-23 odd - Related Rates
2.6 # 25, 27, 35 - Related Rates (more challenging problems)

3.1 # 17-31 odd - Absolute Extrema on an Interval
3.2 # 7-19 odd - Rolle's Theorem
3.2 # 31-37 odd - Mean Value Theorem
3.3 # 11-31 odd - Increasing, Decreasing, and Relative Extrema
3.4 #11-25 odd - Inlection Points and Concavity