1. Find the average rate of change of the volume of a sphere with respect to radius, as the radius of the sphere changes from 1 cm to 2 cm.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{\Delta V}{\Delta r} = \frac{\frac{4}{3}\pi(2)^3 - \frac{4}{3}\pi(1)^3}{2-1}$$

$$= \frac{32\pi}{3} - \frac{4\pi}{3} = \boxed{\frac{28\pi}{3}\text{ cm}^2}$$

2. Find the instantaneous rate of change of the volume of a sphere with respect to radius when the radius is 2 cm.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dr} = 4\pi r^2 \cdot \frac{dr}{dr}$$

$$= 4\pi(2)^2 = \boxed{16\pi\text{ cm}^2}$$

3. If the radius of a sphere changes at a rate of 3cm per second, find the rate of change of the volume of the sphere with respect to time when the radius is 2 cm.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$$

$$= 4\pi(2)^2 \cdot 3 = \boxed{48\pi \text{ cm}^3/\text{s}}$$

4. $x^3 + y^3 - 6xy = 0$

$$3x^2 + 3y^2 \cdot y' - 6(y + xy') = 0$$

$$3x^2 + 3y^2 y' - 6y - 6xy' = 0$$

$$3y^2 y' - 6xy' = 6y - 3x^2$$

$$y'(3y^2 - 6x) = 6y - 3x^2$$

$$y' = \frac{6y - 3x^2}{3y^2 - 6x} = \boxed{\frac{2y - x^2}{y^2 - 2x}}$$

5. $y = \sin(xy)$

$$y' = \cos(xy)\cdot(y + xy')$$

$$y' = y\cos xy + xy'\cos xy$$

$$y' - xy'\cos xy = y\cos xy$$

$$y'(1 - x\cos xy) = y\cos xy$$

$$y' = \boxed{\frac{y\cos xy}{1 - x\cos xy}}$$

Bonus: Determine if the Mean Value Theorem applies to the function on the given interval, and if so, find all values of c guaranteed by the theorem.

$f(x) = \sqrt{2-x}$, $[-7,2]$

f cts on $[-7,2]$? yes
f diff on $(-7,2)$? yes

MVT applies

$$\frac{f(2)-f(-7)}{2-(-7)} = \frac{\sqrt{2-2} - \sqrt{2-(-7)}}{9} = \frac{0-\sqrt{9}}{9} = \frac{-3}{9} = \frac{-1}{3}$$

$$f(x) = (2-x)^{1/2}$$

$$f'(x) = \frac{1}{2}(2-x)^{-1/2}\cdot(-1) = \frac{-1}{2\sqrt{2-x}}$$

$$3x \cdot \frac{1}{x} = \frac{2}{3}\cdot 3x$$

$$3 = 2x$$

$$\frac{-1}{2\sqrt{2-x}} = \frac{-1}{3}$$

$$3 = 2\sqrt{2-x}$$

$$\frac{3}{2} = \sqrt{2-x}$$

$$\frac{9}{4} = 2-x$$

$$x = \frac{8}{4} - \frac{9}{4} = \boxed{\frac{-1}{4}}$$

3.3-3.4 Increasing, Decreasing, Concavity, and the 1st and 2nd Derivative Tests

What do f' and f'' tell us about f?

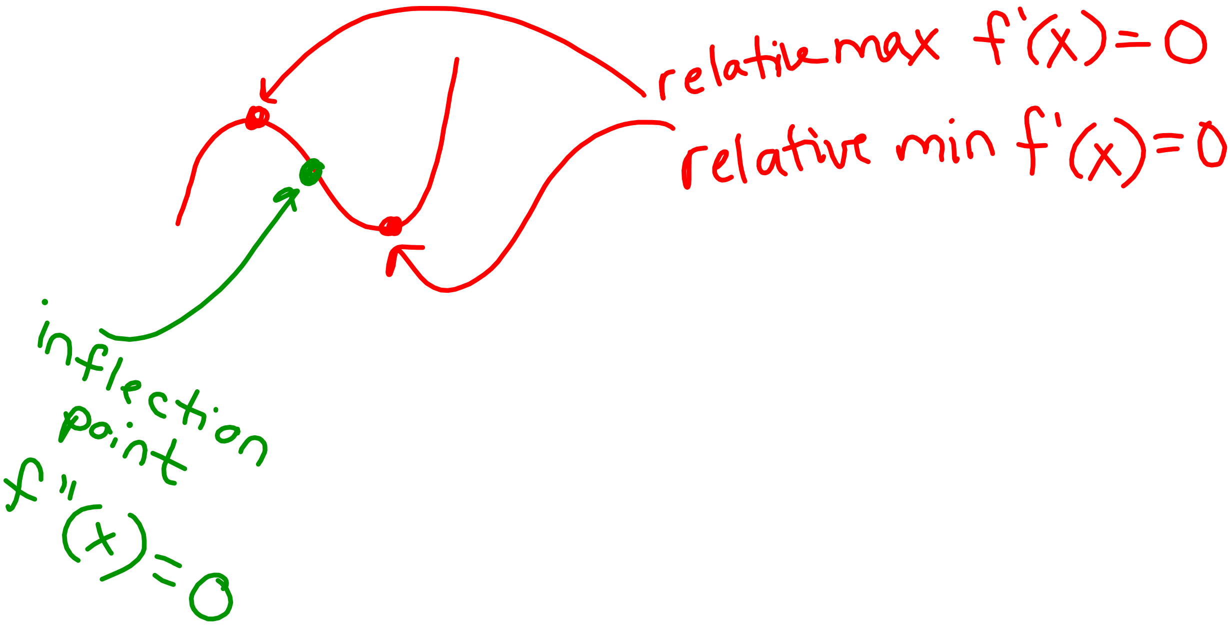
Recall that f' is the rate of change or slope of f,
f'' is the slope or rate of change of f'.

f'	f
+	increasing
−	decreasing

f''	f'	f
+	increasing	concave up
−	decreasing	concave down

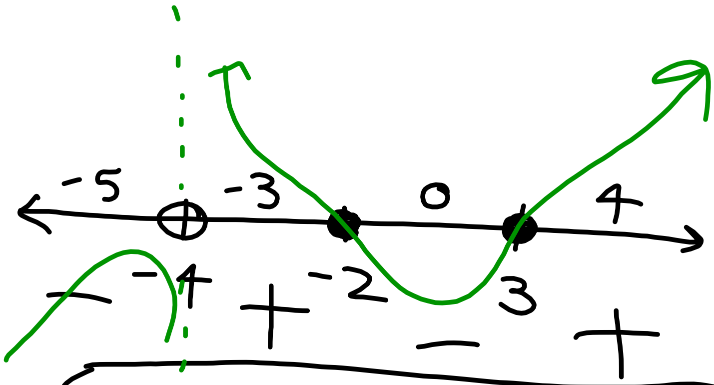
$f'(x)=0$ when f has a relative maximum or minimum.
These x-values (and those where f'(x) is undeined) are called critical numbers.

$f''(x)=0$ when f changes concavity.
The points where concavity changes are called inlection points .



To solve problems involving concavity, increasing/decreasing, etc., we should recall how to solve polynomial inequalities.

$$\frac{(x+2)(x-3)}{x+4} \geq 0$$



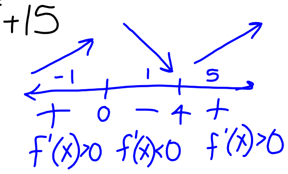
$$\boxed{(-4,-2] \cup [3,\infty)}$$

- Find all critical numbers and state the open intervals on which f is increasing and/or decreasing.
- Find all inflection points and state the open intervals on which f is concave up and/or concave down.
- Use these results to determine all relative and absolute extrema.

3.3

16. $f(x)=x^3-6x^2+15$

$f'(x)=3x^2-12x$
$=3x(x-4)$

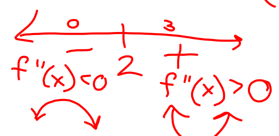


critical #'s:
0, 4

f is increasing on $(-\infty,0)\cup(4,\infty)$
f is decreasing on $(0,4)$
relative max @ $(0,f(0))=(0,15)$
relative min @ $(4,f(4))=(4,-17)$

$f''(x)=6x-12$
$=6(x-2)$

inflection point @ $(2,f(2))=(2,-1)$



f is concave down on $(-\infty,2)$
f is concave up on $(2,\infty)$

3.4

16. $f(x) = x^3(x-4) = x^4 - 4x^3$

$f'(x) = 4x^3 - 12x^2 = 4x^2(x-3)$

critical #'s: 0, 3

$f'(-1)$, $f'(1)$, $f'(4)$

$-$ 0 $-$ 3 $+$

f is decreasing on $(-\infty, 3)$

f is increasing on $(3, \infty)$

$f''(x) = 12x^2 - 24x$

$= 12x(x-2)$

f has inflection points @ $(0, 0)$ & $(2, -16)$

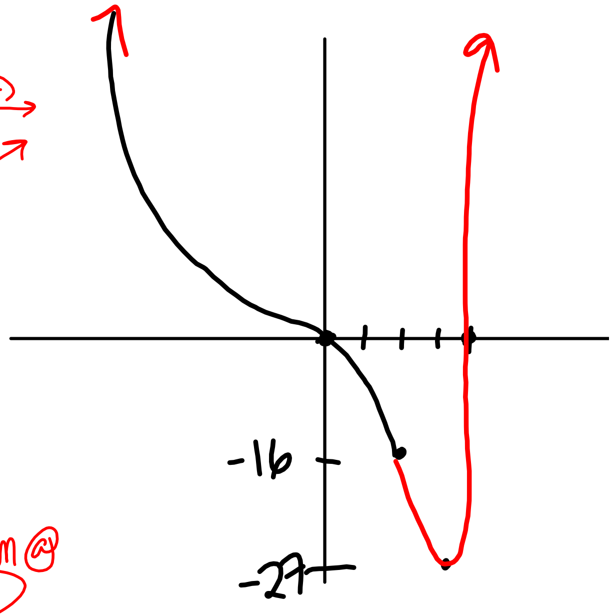
$f''(-1)$, $f''(1)$, $f''(3)$

$+$ 0 $-$ 2 $+$

f has a minimum @ $(3, -27)$

f is concave up on $(-\infty, 0) \cup (2, \infty)$

f is concave down on $(0, 2)$



Homework since Test #2 (Material for Test #3)

2.5 # 1-39 odd; 43, 47 - Implicit Differentiation
2.6 # 15-23 odd - Related Rates
2.6 # 25, 27, 35 - Related Rates (more challenging problems)

3.1 # 17-31 odd - Absolute Extrema on an Interval
3.2 # 7-19 odd - Rolle's Theorem
3.2 # 31-37 odd - Mean Value Theorem
3.3 # 11-31 odd - Increasing, Decreasing, and Relative Extrema
3.4 #11-25 odd - Inlection Points and Concavity