Review: Find the derivative.

$$f(x) = 5^{\csc x}\sqrt{x^3 - 7x}$$

$$f'(x) = \left[5^{\csc x}\right]' \cdot \sqrt{x^3-7x} + \left[(x^3-7x)^{1/2}\right]' \cdot 5^{\csc x}$$

$$= \boxed{5^{\csc x} \cdot \ln 5 \cdot (-\csc x \cot x)\sqrt{x^3-7x} + \frac{1}{2}(x^3-7x)^{-1/2} \cdot (3x^2-7) \cdot 5^{\csc x}}$$

44. $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$ $= 1^\infty$

indeterminate form

$y = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$

$\ln y = \ln\left[\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x\right]$

$\ln y = \lim_{x\to\infty}\left[\ln\left(1+\frac{1}{x}\right)^x\right]$

$\ln y = \lim_{x\to\infty}\left[x \cdot \underbrace{\ln\left(1+\frac{1}{x}\right)}\right]$

$\infty \cdot 0 \Rightarrow$ indeterminate form

$\ln y = \lim_{x\to\infty}\left[\underbrace{\frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}}}\right]$

$\frac{0}{0} \Rightarrow$ L'Hopital's Rule applies

$\ln y = \lim_{x\to\infty} \frac{\frac{1}{1+\frac{1}{x}} \cdot \cancel{(-x^{-2})}}{\cancel{-x^{-2}}}$ ($\frac{1}{x} \to 0$)

$\ln y = 1$

$e^{\ln y} = e^1$

$\boxed{y = e}$

$$\sqrt{\lim_{x\to 2}(x+7)} = \sqrt{2+7} = \sqrt{9} = 3$$

$$\parallel$$

$$\lim_{x\to 2}\sqrt{x+7} = \sqrt{2+7} = \sqrt{9} = 3$$

50. $\lim_{x\to 0^+}\left[\cos\left(\frac{\pi}{2}-x\right)\right]^x$ 0^0 indeterminate

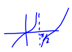
$y = \lim_{x\to 0^+}\left[\cos\left(\frac{\pi}{2}-x\right)\right]^x$

$\ln y = \lim_{x\to 0^+}\left[\underbrace{x\cdot\ln\left(\cos\left(\frac{\pi}{2}-x\right)\right)}_{0\cdot(-\infty)}\right]$

$\ln y = \lim_{x\to 0^+}\frac{\ln\left(\cos\left(\frac{\pi}{2}-x\right)\right)}{\frac{1}{x} = x^{-1}}$ $\Big\}\ \frac{-\infty}{\infty} \Rightarrow$ L'H applies

$\ln y = \lim_{x\to 0^+}\frac{\frac{1}{\cos\left(\frac{\pi}{2}-x\right)}\cdot\left(-\sin\left(\frac{\pi}{2}-x\right)\right)\cdot(-1)}{-\frac{1}{x^2} = -x^{-2}}$

$\ln y = \lim_{x\to 0^+}\frac{\tan\left(\frac{\pi}{2}-x\right)}{\frac{-1}{x^2} = -x^{-2}}$ $\Big\}\ \frac{\infty}{\infty}$



$\ln y = \lim_{x\to 0^+}\frac{\sec^2\left(\frac{\pi}{2}-x\right)\cdot(-1)}{\frac{2}{x^3} = 2x^{-3}}$ $\Big\}\ \frac{\infty}{\infty}$

$\ln y = \lim_{x\to 0^+}\frac{-2\sec\left(\frac{\pi}{2}-x\right)\cdot\sec\left(\frac{\pi}{2}-x\right)\tan\left(\frac{\pi}{2}-x\right)\cdot(-1)}{\frac{-6}{x^4} = -6x^{-4}}$

$\ln y = \lim_{x\to 0}\frac{-\sec^2\left(\frac{\pi}{2}-x\right)\tan\left(\frac{\pi}{2}-x\right)}{\frac{3}{x^4}}$

$\ln y = \lim_{x\to 0^+}\frac{-\sec^2\left(\frac{\pi}{2}-x\right)}{\frac{2}{x^3}}\cdot\frac{\tan\left(\frac{\pi}{2}-x\right)}{\frac{3}{2x}}$

$\ln y = \underbrace{\lim_{x\to 0^+}\left(\frac{-\sec^2\left(\frac{\pi}{2}-x\right)}{\frac{2}{x^3}}\right)}_{\ln y}\lim_{x\to 0^+}\frac{\tan\left(\frac{\pi}{2}-x\right)}{\frac{3}{2x}}$

$\ln y = \ln y\left[\lim_{x\to 0^+}\frac{\tan\left(\frac{\pi}{2}-x\right)}{\frac{3}{2x}}\right]$

$\ln y = \ln y\left[\lim_{x\to 0^+}\frac{\tan\left(\frac{\pi}{2}-x\right)}{\frac{-1}{x^3}}\cdot\frac{\frac{-1}{3x^2}}{2}\right]$

$\ln y = (\ln y)^2\cdot\lim_{x\to 0^+}\frac{-2}{3x^2}$

$0 = (\ln y)^2\cdot\lim_{x\to 0^+}\left(\frac{-2}{3x^2}\right) - \ln y$

$0 = \ln y\left[\ln y\cdot\lim_{x\to 0^+}\left(\frac{-2}{3x^2}\right) - 1\right]$

$\ln y = 0$ $\qquad \ln y = \frac{1}{\lim_{x\to 0^+}\frac{-2}{3x^2}}$

$e^{\ln y} = e^0$ $\qquad \ln y = 0$

$\boxed{y = 1}$

3.7 Optimization Problems

4. Find two positive numbers whose product is 192 and the sum of the irst plus three times the second is a minimum.

$x \cdot y = 192$

$S = x + 3y$

$S = \frac{192}{y} + 3y$

$S' = \frac{-192}{y^2} + 3$

$\frac{-192}{y^2} + 3 = 0$

$\frac{-192}{y^2} = -3$

$\frac{192}{3} = y^2$

$\sqrt{\frac{192}{3}} = y$

$8 = y$

$3\overline{)192}$ = 64

$x = \frac{192}{8} = 24$

Homework:

Limits at Ininity: 3.5 #15-31 odd
L'Hopital's Rule: 7.7 #11-35 odd
L'Hopital's Rule with logs: 7.7 #37-53odd
Optimization: 3.7 #3,5,17,23,29