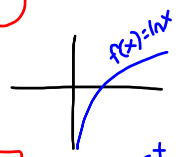
$$\lim_{x\to 0^+} (\sin x)^x$$

0^0



as $x\to 0^+$
$\ln x \to -\infty$

$$y = \lim_{x\to 0^+} (\sin x)^x$$

$$\ln y = \lim_{x\to 0^+} \left[x \ln(\sin x) \right]$$

$$\ln y = \lim_{x\to 0^+} \frac{\ln(\sin x)}{\frac{1}{x}} \qquad \frac{-\infty}{\infty}$$

$$\ln y = \lim_{x\to 0^+} \frac{\frac{1}{\sin x}\cdot \cos x}{\frac{-1}{x^2}}$$

$$\frac{\cos x}{\sin x}\cdot \frac{-1}{\frac{1}{x}\cdot\frac{1}{x}}$$

$$\frac{\cos x}{\sin x}\cdot \frac{-1\cdot\frac{x}{1}}{\frac{1}{x}}$$

$$\ln y = \lim_{x\to 0^+} \frac{\overset{0}{-x}\ \overset{1}{\cos x}}{\underset{1}{\frac{\sin x}{x}}}$$

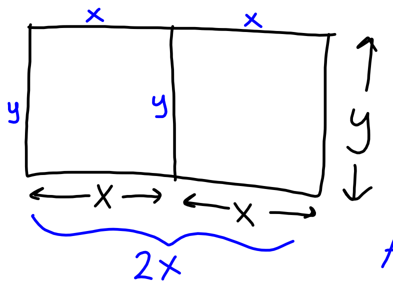
$$\ln y = 0$$

$$e^{\ln y} = e^0$$

$$\boxed{y = 1}$$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1 \ ; \ \lim_{x\to 0} \frac{1-\cos x}{x} = 0$$

18. A rancher has 200 feet of fencing with which to enclose two adjacent corrals, arranged according to the igure. What dimensions should be used so that the enclosed area will be a maximum?



$$200 = 3y + 4x$$

$$A = 2xy$$

$$A = 2y\left(50 - \tfrac{3}{4}y\right)$$

$$A = 100y - \tfrac{3}{2}y^2$$

$$A' = 100 - 3y$$

$$100 - 3y = 0$$

$$-3y = -100 \qquad y = \frac{100}{3}$$

$$4x = 200 - 3y$$

$$x = 50 - \tfrac{3}{4}y$$

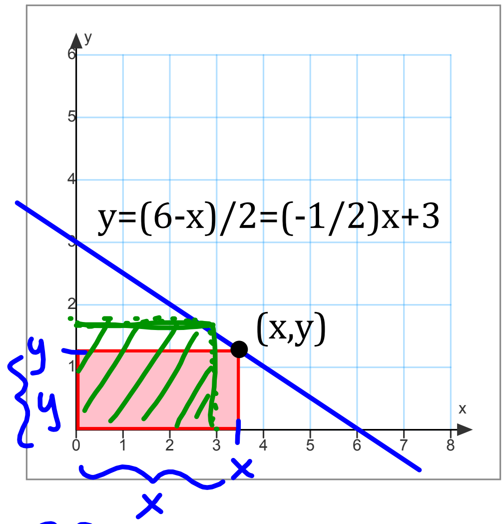
$$x = 50 - \frac{\cancel{3}}{\cancel{4}}\left(\frac{\overset{25}{\cancel{100}}}{\cancel{3}}\right)$$

$$= 25$$

Dimensions which maximize area:

$x = 25$ ft ; $y = \frac{100}{3}$ ft

24. A rectangle is bounded by the x- and y-axes and the graph of y=(6-x)/2. What length and width should the rectangle have so that its area is a maximum?



$$A = x\left(-\tfrac{1}{2}x + 3\right)$$

$$A = -\tfrac{1}{2}x^2 + 3x$$

$$A' = -x + 3$$

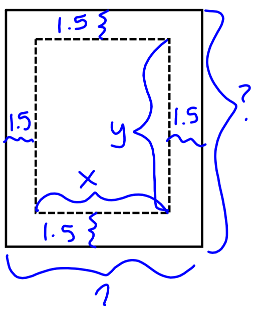
$$-x + 3 = 0$$

area $= x \cdot y$

$$y = -\tfrac{1}{2}(3) + 3 = \tfrac{3}{2}$$

$$\boxed{x = 3 \ ; \ y = \tfrac{3}{2}}$$

30. A rectangular page is to contain 36 square inches of print. The margins on each side are to be 1.5 inches. Find the dimensions of the page such that the least amount of paper is used.



$$36 = xy \Rightarrow y = \frac{36}{x}$$

$$A = (x+3)(y+3)$$

$$A = (x+3)\left(\frac{36}{x} + 3\right)$$

$$A = 36 + 3x + \frac{3 \cdot 36}{x} + 9$$

$$A = 45 + 3x + 108x^{-1}$$

$$A' = 3 - 108x^{-2}$$

$$3 - \frac{108}{x^2} = 0$$

$$\frac{3}{1} = \frac{108}{x^2}$$

$$3x^2 = 108$$

$$x^2 = \frac{\cancel{3} \cdot 36}{\cancel{3}}$$

$$x = 6 \ ; \ y = 6$$

Dimensions of paper : $\boxed{9 \text{in} \times 9 \text{in}}$

Homework:

Limits at Ininity: 3.5 #15-31 odd
L'Hopital's Rule: 7.7 #11-35 odd
L'Hopital's Rule with logs: 7.7 #37-53odd
Optimization: 3.7 #3,5,17,23,29