

1. Use the product rule to differentiate the function.

a. $f(x) = (x^2 + \sin x)(3x - 15)$

b. $f(x) = \sqrt{x} \tan x$

2. Use the quotient rule to differentiate the function.

a. $f(x) = \frac{\cos x}{x^2}$

b. $f(x) = \frac{2(\cos x - 1)}{3 \sin x}$

3. Find $f'(3)$ given that $g(3) = 2$, $g'(3) = -1$, $h(3) = 5$, and $h'(3) = -2$

a. $f(x) = 2g(x)h(x)$

b. $f(x) = 2g(x) + h(x)$

c. $f(x) = 7 + \frac{g(x)}{h(x)}$

4. Find the second derivative of the function.

a. $f(x) = \frac{\tan x - 2}{3}$

b. $f(x) = \csc^2 \pi x$

1. Use the product rule to differentiate the function.

a. $f(x) = (x^2 + \sin x)(3x - 15)$

b. $f(x) = \sqrt{x} \tan x$

$$\begin{aligned} \text{a. } f'(x) &= (x^2 + \sin x)'(3x - 15) + (x^2 + \sin x)(3x - 15)' \\ &= (2x + \cos x)(3x - 15) + (x^2 + \sin x)(3) \end{aligned}$$

$$\begin{aligned} \text{b. } f'(x) &= (x^{1/2})' \tan x + \sqrt{x} \cdot (\tan x)' \\ &= \frac{1}{2} x^{-1/2} \tan x + \sqrt{x} \cdot \sec^2 x \end{aligned}$$

2. Use the quotient rule to differentiate the function.

a. $f(x) = \frac{\cos x}{x^2}$

b. $f(x) = \frac{2(\cos x - 1)}{3 \sin x}$

$$\text{a. } f'(x) = \frac{x^2(-\sin x) - (\cos x)(2x)}{(x^2)^2}$$

$$\text{b. } f'(x) = \frac{(3 \sin x)(-2 \sin x) - 2(\cos x - 1)(3 \cos x)}{(3 \sin x)^2}$$

3. Find $f'(3)$ given that $g(3) = 2$, $g'(3) = -1$, $h(3) = 5$, and $h'(3) = -2$

a. $f(x) = 2g(x)h(x)$

b. $f(x) = 2g(x) + h(x)$

c. $f(x) = 7 + \frac{g(x)}{h(x)}$

a. $f'(x) = 2g'(x)h(x) + 2g(x)h'(x)$

$$f'(3) = 2(-1)(5) + 2(2)(-2) \\ = -10 - 8 = \boxed{-18}$$

b. $f'(x) = 2g'(x) + h'(x)$

$$f'(3) = 2(-1) + (-2) = \boxed{-4}$$

c. $f'(x) = \frac{h(x)g'(x) - g(x)h'(x)}{[h(x)]^2}$

$$f'(3) = \frac{5(-1) - 2(-2)}{5^2} = \frac{-5 + 4}{25} = \boxed{\frac{-1}{25}}$$

4. Find the second derivative of the function.

a. $f(x) = \frac{\tan x - 2}{3} = \frac{1}{3}\tan x - \frac{2}{3}$

b. $f(x) = \csc^2 \pi x = [\csc(\pi x)]^2$

a. $f'(x) = \frac{1}{3}\sec^2 x = \frac{1}{3}[\sec x]^2$

$$f''(x) = \boxed{\frac{2}{3}\sec x \cdot \sec x \tan x}$$

b. $f'(x) = 2\csc(\pi x) \cdot [-\csc \pi x \cot \pi x] \cdot \pi$

$$= -2\pi \csc^2(\pi x) \cot(\pi x)$$

$$f''(x) = -2\pi (\csc^2 \pi x)' \cot \pi x + (-2\pi) \csc^2 \pi x (\cot \pi x)'$$

$$= -2\pi (-2\pi \csc^2 \pi x \cot \pi x) \cot \pi x - 2\pi \csc^2 \pi x (-\csc^2 \pi x) \cdot \pi$$

$$= \boxed{4\pi^2 \csc^2 \pi x \cot^2 \pi x + 2\pi^2 \csc^4 \pi x}$$