5. Find the derivative of the function.

a. $f(x) = \sqrt{\frac{x}{x^3-2x}}$

b. $f(x) = \left(\frac{\sin x}{x^3-2x}\right)^3$

c. $f(x) = 3x - 5\cot(\pi x)^2$

d. $f(x) = \ln(\tan^{-1}(2x))$

e. $f(x) = 5^{\csc x}\sqrt{x^3 - 7x}$

$\tan^{-1}x = \arctan x$

6. Find an equation of the tangent line to the graph of f at the indicated point.

$f(x) = \sqrt{2x^2 - 7}$, $(2, 1)$

$y - y_1 = m(x - x_1)$

$f'(x) = ?$

$m = f'(2)$

7. The length of a rectangle is given by $2(t^2 + 1)$ and its height is $\sqrt{t+5}$ where t is time in seconds and dimensions are in centimeters.

$A(t) =$

a. Find the average rate of change of the area from time 4 to time 11.

$\frac{\Delta A}{\Delta t} = \frac{A(11) - A(4)}{11 - 4}$

b. Find the instantaneous rate of change of the area at time 4.

$A'(4)$

5. Find the derivative of the function.

a. $f(x) = \sqrt{\frac{x}{x^3-2x}}$

b. $f(x) = \left(\frac{\sin x}{x^3-2x}\right)^3$

$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

a. $f(x) = \left(\frac{x}{x^3-2x}\right)^{1/2}$

$f'(x) = \boxed{\frac{1}{2}\left(\frac{x}{x^3-2x}\right)^{-1/2} \cdot \frac{1(x^3-2x) - x(3x^2-2)}{(x^3-2x)^2}}$

b. $f'(x) = \boxed{3\left(\frac{\sin x}{x^3-2x}\right)^2 \cdot \frac{\cos x(x^3-2x) - \sin x(3x^2-2)}{(x^3-2x)^2}}$

c. $f(x) = 3x - 5\cot(\pi x)^2$

d. $f(x) = \ln(\tan^{-1}(2x))$

e. $f(x) = 5^{\csc x}\sqrt{x^3 - 7x}$

c. $f(x) = 3x - 5\cot(\pi^2 x^2)$

$f'(x) = \boxed{3 - 5(-\csc^2(\pi^2 x^2)) \cdot 2\pi^2 x}$

d. $f(x) = \ln[\arctan(2x)]$

$f'(x) = \boxed{\frac{1}{\arctan 2x} \cdot \frac{1}{1+(2x)^2} \cdot 2}$

e. $f(x) = (5^{\csc x})((x^3-7x)^{1/2})$

$f'(x) = \boxed{5^{\csc x} \cdot \ln 5 \, (-\csc x \cot x) \cdot (x^3-7x)^{1/2} + 5^{\csc x} \cdot \frac{1}{2}(x^3-7x)^{-1/2} \cdot (3x^2-7)}$

6. Find an equation of the tangent line to the graph of f at the indicated point.

$f(x) = \sqrt{2x^2 - 7}$, $(2, 1)$

$f(x) = (2x^2 - 7)^{1/2}$

$f'(x) = \frac{1}{\cancel{2}}(2x^2-7)^{-1/2} \cdot \overset{2}{\cancel{4}}x$

$m = f'(2) = \frac{2(2)}{\sqrt{2(2)^2 - 7}} = \frac{4}{\sqrt{8-7}} = 4$

$y - y_1 = m(x - x_1)$

$y - 1 = 4(x - 2)$

$y = 4x - 8 + 1$

$\boxed{y = 4x - 7}$

7. The length of a rectangle is given by $2(t^2+1)$ and its height is $\sqrt{t+5}$ where t is time in seconds and dimensions are in centimeters.

$$A(t) = 2(t^2+1)\cdot\sqrt{t+5}$$
$$= (2t^2+2)(t+5)^{1/2}$$

a. Find the average rate of change of the area from time 4 to time 11.

$$\frac{\Delta A}{\Delta t} = \frac{A(11)-A(4)}{11-4} = \frac{2(11^2+1)\cdot\sqrt{11+5} - 2(4^2+1)\cdot\sqrt{4+5}}{7}$$
$$= \frac{8(122)-6(17)}{7} = \boxed{\frac{874}{7}\ \mathrm{cm}^2/\mathrm{s}}$$

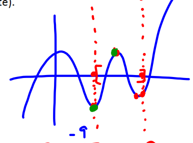
b. Find the instantaneous rate of change of the area at time 4.

$$A'(t) = 4t\sqrt{t+5} + (2t^2+2)\cdot\frac{1}{2\sqrt{t+5}}$$
$$A'(4) = 16(3) + (34)\cdot\frac{1}{6} = 48+\frac{17}{3} = \boxed{\frac{161}{3}\ \mathrm{cm}^2/\mathrm{s}}$$

$$\begin{array}{r} 48 \\ \times 3 \\ \hline 144 \\ +17 \\ \hline 161 \end{array}$$

2. Identify the absolute maximum value and absolute minimum value of the function on the closed interval, and the x-values at which they occur. Give exact values (as fractions, as appropriate).

$f(x) = \frac{2}{3}x^3 + \frac{1}{2}x^2 - 6x + 1$; $[-3,2]$



$f'(x) = 2x^2+x-6$

critical #'s

$2x^2+x-6=0$

$2x^2+4x-3x-6=0$

$2x(x+2)-3(x+2)=0$

$(x+2)(2x-3)=0$

$x = -2 \ \& \ 3/2$

both in $[-3,2]$

$f(-3) = \frac{2}{\cancel{3}}\cdot\frac{\cancel{-27}^{-9}}{1} + \frac{1}{2}\cdot\frac{9}{1} + 18 + 1$

$= -18 + \frac{9}{2} + 18 + \frac{2}{2} = \boxed{\frac{11}{2}}$

$f(-2) = \frac{2}{3}\cdot\frac{-8}{1} + \frac{1}{2}\cdot\frac{4}{1} + 12 + 1$

$= -\frac{16}{3} + 2 + 12 + 1$

$= -\frac{16}{3} + \frac{45}{3} = \boxed{\frac{29}{3}}$

$f(3/2) = \frac{2}{3}\left(\frac{3}{2}\right)^3 + \frac{1}{2}\left(\frac{3}{2}\right)^2 - 6\left(\frac{3}{2}\right) + 1$

$= \frac{\cancel{2}}{\cancel{3}}\cdot\frac{\cancel{27}^{9}}{\cancel{8}_{4}} + \frac{1}{2}\cdot\frac{9}{4} - 9 + 1$

$= \frac{9}{4} + \frac{9}{8} - 8 = \frac{18}{8} + \frac{9}{8} - \frac{64}{8}$

$= \boxed{\frac{-37}{8}}$

$f(2) = \frac{2}{3}(2)^3 + \frac{1}{2}(2)^2 - 6(2) + 1$

$= \frac{2}{3}\cdot\frac{8}{1} + \frac{1}{2}\cdot\frac{4}{1} - 12 + 1$

$= \frac{16}{3} + 2 - 12 + 1 = \frac{16}{3} - 9$

$= \frac{16}{3} - \frac{27}{3} = \boxed{\frac{-11}{3}}$

Absolute maximum: $\frac{29}{3}$ @ $x=-2$

Absolute Minimum: $-\frac{37}{8}$ @ $x = 3/2$

3. Find the critical numbers of f (if any), state the open intervals on which f is increasing and decreasing, and identify any relative maximum or minimum points.

$f(x) = \dfrac{x}{(1+x)^2}$

$$f'(x) = \frac{1(1+x)^2 - x\cdot 2(1+x)}{(1+x)^4} = \frac{1+2x+x^2-2x-2x^2}{(1+x)^4}$$

$$= \frac{1-x^2}{(1+x)^4}$$

$f'(x) = 0$ @ $x = 1$

$f'(x)$ is undefined @ $x = -1$

$f'(-2)$ $f'(0)$ $f'(2)$

$-$ $\;-1\;$ $+$ $\;1\;$ $-$

$$\boxed{\begin{array}{l} \text{critical \#'s: } 1 \;\&\; {-1} \\ f \text{ is increasing} \\ \quad \text{on } (-1,1) \\ f \text{ is decreasing} \\ \quad \text{on } (-\infty,-1)\cup(1,\infty) \\ \text{relative maximum} \\ @\left(1,\tfrac{1}{4}\right) \end{array}}$$

4. Find the points of inflection of f (if any), and discuss the concavity of the graph of the function.

$f(x) = x^3 - 12x - 16 = (x-4)(x+2)^2$

$f'(x) = 3x^2 - 12$

$f''(x) = 6x$

$6x = 0$

$x = 0$

$f''(-1)$ $f''(1)$

$-$ $\;0\;$ $+$

$$\boxed{\begin{array}{l} f \text{ is concave down} \\ \quad \text{on } (-\infty,0) \\ f \text{ is concave up} \\ \quad \text{on } (0,\infty) \\ f \text{ has an inflection point} \\ \quad (0,-16) \end{array}}$$

Questions about textbook problems on l'Hopital's rule or optimization?
(I will assign homework grades Friday based on assignments from 2.5 through 3.7)

3.7 #23 tomorrow

Old Homework (topics for inal exam)

Limits (Test 1)

- 1.2 #1-7odd,9-18all — Finding limits graphically and numerically
- 1.2 #23, 25, 27, 29; 30, 31 — Epsilon-Delta deinition of the limit
- 1.3 #11, 17, 27-35 odd; 39 — Evaluating limits analytically
- 1.4 #7-17odd;25-28all;39-47odd;57,59 — Discontinuity and one-sided limits
- 1.4 #19,21,23,51,63,69,71,83,85 — Continuity with Trig and Intermediate Value Theorem
- 1.5 #1-51 odd — Ininite limits
- Ch 1 review pp. 88-89

Derivative Basics (Test 2)

- 2.1 #1-23odd — Find the derivative by the limit process
- 2.1 #29-32 — Find the equation of the tangent line
- 2.1 #61-69 odd — Use the alternate form to ind the derivative
- 2.1 #71-79 odd — Describe the x-values where the function is differentiable (given a graph)
- 2.2 #3-51 odd — Find derivative using basic rules
- 2.2 #91-94; 101,102 — Use derivative to solve rate of change word problems
- 2.3 #1-53odd,63-69odd, 75-81all,83-91odd,109-115all — Product and quotient rules
- 2.4 #7-81odd — Chain rule
- 5.1 #45-61, 71 — Logarithmic functions
- 5.4 #39-57 — Exponential functions
- 5.5 #41-55 — Log and exp functions with other bases
- 5.8 #41-59 — Inverse trig functions

Derivative Applications (Test 3)

- 2.5 # 1-39 odd; 43, 47 — Implicit Differentiation
- 2.6 # 15-23 odd — Related Rates
- 2.6 # 25, 27, 35 — Related Rates (more challenging problems)
- 3.1 # 17-31 odd — Absolute Extrema on an Interval
- 3.2 # 7-19 odd — Rolle's Theorem
- 3.2 # 31-37 odd — Mean Value Theorem
- 3.3 # 11-31 odd — Increasing, Decreasing, and Relative Extrema
- 3.4 #11-25 odd — Inlection Points and Concavity

Limits, Derivatives and Applications (Since Test 3)

- 3.5 #15-31 odd — Limits at Ininity
- 7.7 #11-35 odd — L'Hopital's Rule
- 7.7 #37-53 odd — L'Hopital's Rule with logs
- 3.7 #3,5,17,23,29 — Optimization