Homework (topics for inal exam - BRING 2.5 through 3.7 to inal exam!)

Limits (Test 1)

- 1.2 #1-7odd,9-18all — Finding limits graphically and numerically
- 1.2 #23, 25, 27, 29; 30, 31 — Epsilon-Delta deinition of the limit
- 1.3 #11, 17, 27-35 odd; 39 — Evaluating limits analytically
- 1.4 #7-17odd;25-28all;39-47odd;57,59 — Discontinuity and one-sided limits
- 1.4 #19,21,23,51,63,69,71,83,85 — Continuity with Trig and Intermediate Value Theorem
- 1.5 #1-51 odd — Ininite limits
- Ch 1 review pp. 88-89

Derivative Basics (Test 2)

- 2.1 #1-23odd — Find the derivative by the limit process
- 2.1 #29-32 — Find the equation of the tangent line
- 2.1 #61-69 odd — Use the alternate form to ind the derivative
- 2.1 #71-79 odd — Describe the x-values where the function is differentiable (given a graph)
- 2.2 #3-51 odd — Find derivative using basic rules
- 2.2 #91-94; 101,102 — Use derivative to solve rate of change word problems
- 2.3 #1-53odd,63-69odd, 75-81all,83-91odd,109-115all — Product and quotient rules
- 2.4 #7-81odd — Chain rule
- 5.1 #45-61, 71 — Logarithmic functions
- 5.4 #39-57 — Exponential functions
- 5.5 #41-55 — Log and exp functions with other bases
- 5.8 #41-59 — Inverse trig functions

Derivative Applications (Test 3)

- 2.5 # 1-39 odd; 43, 47 — Implicit Differentiation
- 2.6 # 15-23 odd — Related Rates
- 2.6 # 25, 27, 35 — Related Rates (more challenging problems)
- 3.1 # 17-31 odd — Absolute Extrema on an Interval
- 3.2 # 7-19 odd — Rolle's Theorem
- 3.2 # 31-37 odd — Mean Value Theorem
- 3.3 # 11-31 odd — Increasing, Decreasing, and Relative Extrema
- 3.4 #11-25 odd — Inlection Points and Concavity

Limits, Derivatives and Applications (Since Test 3)

- 3.5 #15-31 odd — Limits at Ininity
- 7.7 #11-35 odd — L'Hopital's Rule
- 7.7 #37-53 odd — L'Hopital's Rule with logs
- 3.7 #3,5,17,23,29 — Optimization

$$g.\ \lim_{x\to\infty}\frac{e^{x/2}}{x^3} = \lim_{x\to\infty}\frac{\frac{1}{2}e^{x/2}}{3x^2} = \lim_{x\to\infty}\frac{\frac{1}{4}e^{x/2}}{6x}$$

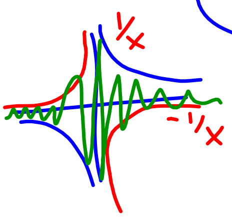
$$= \lim_{x\to\infty}\frac{\frac{1}{8}e^{x/2}}{6} = \boxed{\infty}$$

$$h.\ \lim_{x\to\infty}\frac{\sin x}{x}$$

$$-\frac{1}{x} \le \frac{\sin x}{x} \le \frac{1}{x}$$

$$\lim_{x\to\infty} -\frac{1}{x} \le \lim_{x\to\infty}\frac{\sin x}{x} \le \lim_{x\to\infty}\frac{1}{x}$$

$$0 \qquad\qquad 0$$



By Squeeze Thm

$$\Rightarrow \lim_{x\to\infty}\frac{\sin x}{x} = \boxed{0}$$

$i.\ \lim_{x\to\infty}(1+x)^{1/x}$

$\log_a(x^p) = p\log_a x$

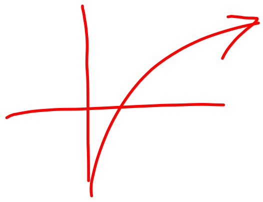
$y = \lim_{x\to\infty}(1+x)^{1/x}$

$\ln y = \ln\left(\lim_{x\to\infty}(1+x)^{1/x}\right)$

$\ln y = \lim_{x\to\infty}\ln(1+x)^{1/x}$

$\ln y = \lim_{x\to\infty}\frac{1}{x}\ln(1+x)$

$\ln y = \lim_{x\to\infty}\frac{\ln(1+x)}{x}$



$\ln y = \lim_{x\to\infty}\frac{\frac{1}{1+x}}{1}$

$\ln y = \lim_{x\to\infty}\frac{1}{1+x}$

$\ln x \to \infty$ as $x \to \infty$

$\ln y = 0$

$e^{\ln y} = e^0$

$e^{\log_e y} = e^0$

$y = 1$

$y = 1$

$a^{\log_a x} = x$

14. Find the derivative of the function using the definition (limit of the difference quotient)
$f(x) = x^3 - 12x$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{(x+h)^3 - 12(x+h) - (x^3-12x)}{h}$$

$$= \lim_{h\to 0}\frac{\cancel{x^3}+3x^2h+3xh^2+h^3-\cancel{12x}-12h-\cancel{x^3}+\cancel{12x}}{h}$$

$$= \lim_{h\to 0}\frac{\cancel{h}(3x^2+3xh+h^2-12)}{\cancel{h}} = \boxed{3x^2-12}$$

18. Determine whether the Mean Value Theorem can be applied to f on the closed interval, and if so, find all values of c such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

$f(x) = \frac{x+1}{x}, \left[\frac{1}{2}, 2\right]$

f is continuous on $\left[\frac{1}{2}, 2\right]$
& differentiable on $\left(\frac{1}{2}, 2\right)$ $\Rightarrow$ MVT applies

$$\frac{f(b)-f(a)}{b-a} = \frac{\frac{2+1}{2} - \frac{\frac{1}{2}+1}{\frac{1}{2}}}{2-\frac{1}{2}} = \frac{\frac{3}{2} - (1+2)}{\frac{3}{2}}$$

$= 1 - 2 = -1$

$$f'(x) = \frac{x(1) - (x+1) \cdot 1}{x^2} = \frac{-1}{x^2}$$

$\frac{-1}{x^2} = -1$

$x^2 = 1$

$x = \pm 1$

only $\boxed{c=1}$ is in $\left(\frac{1}{2}, 2\right)$

9. Discuss the continuity of the function (identify all discontinuities, if any, as removable or non-removable). & state open intervals on which f is cts.

$$f(x) = \frac{x^2 - 7x + 10}{x^2 - 3x + 2} = \frac{(x-2)(x-5)}{(x-2)(x-1)}$$

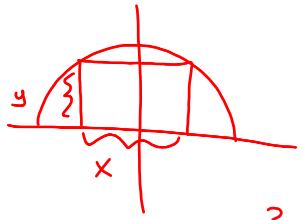
f has removable discontinuities @ 2

& non-removable @ 1

f is continuous on:

$(-\infty, 1) \cup (1, 2) \cup (2, \infty)$

15. A rectangle is bounded by the x-axis and the semi-circle $y = \sqrt{25 - x^2}$
What ~~length and width~~ dimensions should the rectangle have so that its area is a maximum?



$A = xy = x\sqrt{25-x^2} = x\cdot(25-x^2)^{1/2}$

$A' = 1\sqrt{25-x^2} + x\left(\frac{1}{2}(25-x^2)^{-1/2}(-2x)\right)$

$A' = \sqrt{25-x^2} - \frac{x^2}{\sqrt{25-x^2}} = 0$

$\sqrt{25-x^2} = \frac{x^2}{\sqrt{25-x^2}}$

$25-x^2 = x^2$

$25 = 2x^2$

$\frac{25}{2} = x^2$

$x = \frac{5}{\sqrt{2}}$

$y = \sqrt{25-\left(\frac{5}{\sqrt{2}}\right)^2}$

$= \sqrt{\frac{25}{1}\cdot\frac{2}{2} - \frac{25}{2}}$

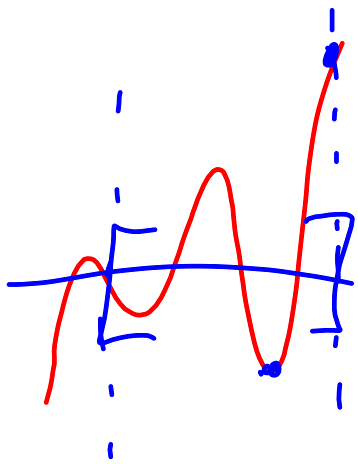
$= \sqrt{\frac{25}{2}}$

$y = \frac{5}{\sqrt{2}}$

it's a

17. Locate the absolute extrema on the closed interval.

$f(x) = x^3 - 12x, \qquad [0,4]$



$f'(x) = 3x^2 - 12$

critical #'s: $3x^2 - 12 = 0$

$3x^2 = 12$

$x^2 = 4$

$x = \pm 2$

$f(2) = 8 - 24$
$= -16$

$f(0) = 0$

$f(4) = 16$

abs max: 16 @ x=4
min: -16 @ x=2

19. Find the critical numbers of f (if any), find the open intervals on which the function is increasing or decreasing, and locate all relative extrema.

$$f(x) = \frac{x^2 - 2x + 1}{x + 1}$$

$$f'(x) = \frac{(x+1)(2x-2) - (x^2-2x+1)(1)}{(x+1)^2}$$

$$= \frac{2x^2 - 2x + 2x - 2 - x^2 + 2x - 1}{(x+1)^2}$$

$$f'(x) = \frac{x^2 + 2x - 3}{(x+1)^2} = \frac{(x+3)(x-1)}{(x+1)^2}$$

critical #'s : $-3, 1, -1$

$f'(-4)$ $f'(-2)$ $f'(0)$ $f'(2)$

$+$ -3 $-$ -1 $-$ 1 $+$

$\nearrow$ $\searrow$ $\searrow$ $\nearrow$

f incr. on $(-\infty, -3) \cup (1, \infty)$

decr. on $(-3, -1) \cup (-1, 1)$

rel max @ $(-3, -8)$; min: $(1, 0)$

20. Find the points of inflection and discuss the concavity of the graph of the function.

$$f(x) = 2x^4 - 8x + 3$$

$f'(x) = 8x^3 - 8$

$f''(x) = 24x^2$

$24x^2 = 0$

$x = 0$

$f''(-1)$ $f''(1)$

$-$ 0 $+$

f is concave up on $(0, \infty)$

& conc. down on $(-\infty, 0)$

f has an inflection point @ $(0, 3)$