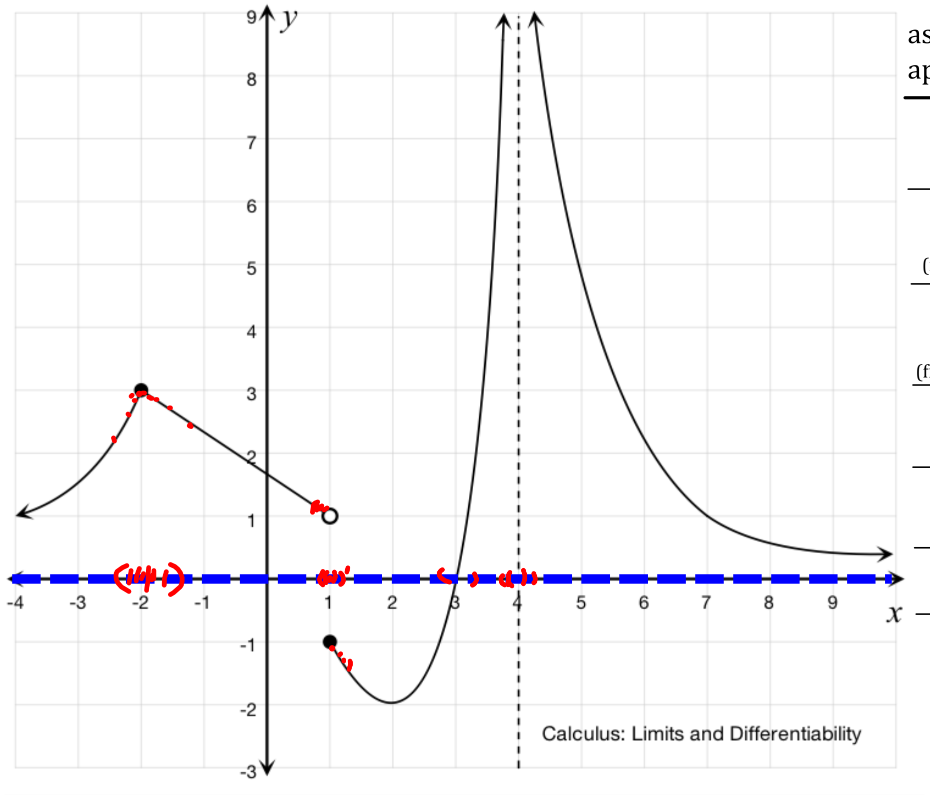
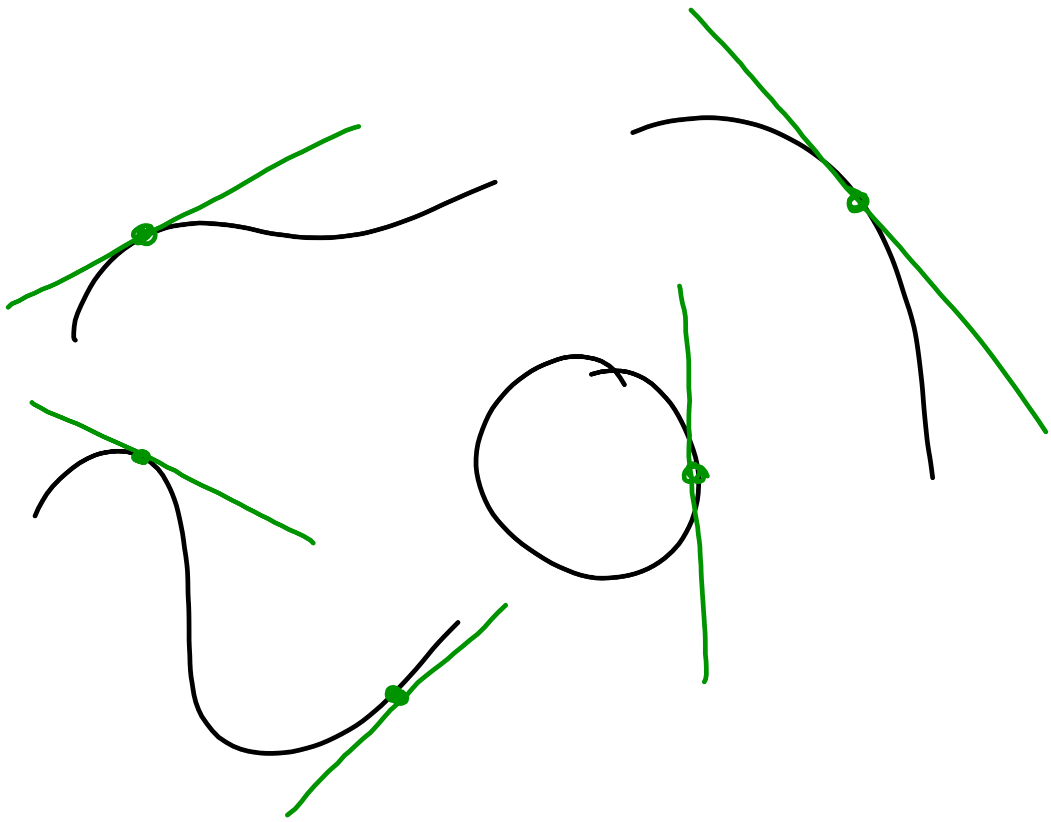
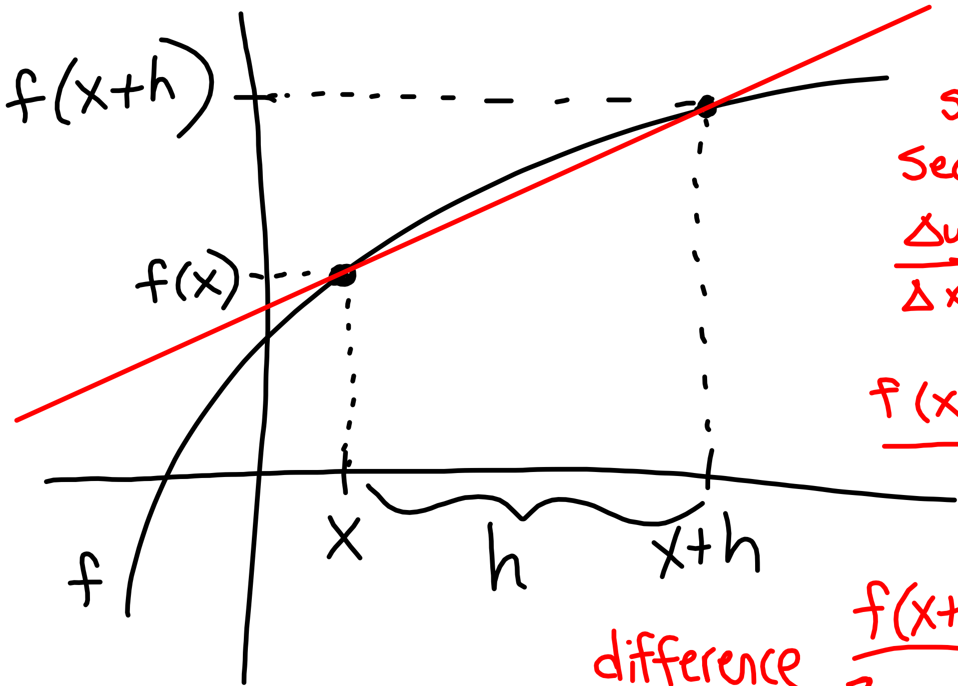


as x approaches...	$f(x)$ approaches...
-2	3
1^- (from the left)	1
1^+ (from the right)	-1
3	0
$-\infty$	0
∞	0
4	∞

tangent lines



secant line



Slope of Secant line:

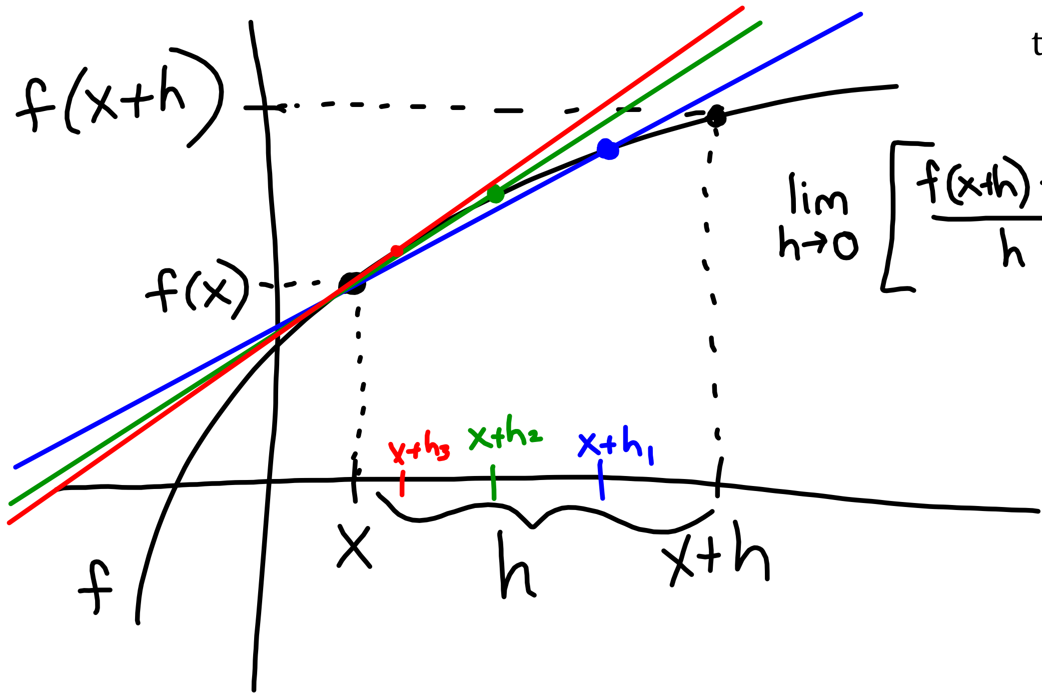
$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{f(x+h) - f(x)}{x+h-x}$$

difference quotient

$$\frac{f(x+h) - f(x)}{h}$$

tangent line



$$\lim_{h \to 0} \left[\frac{f(x+h) - f(x)}{h} \right]$$

Δx "delta x"

change in x

$$\frac{f(x+\Delta x)-f(x)}{\Delta x} = \frac{f(x+h)-f(x)}{h}$$

Δx

Δx

1.2

$$f(x)=\frac{x-2}{x^2-4}, \quad x\neq 2,-2$$

What happens to *f(x)* as *x* approaches 2?

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	0.2564	0.2506	0.25006	0.25	0.2499	0.2494	0.2439

$$\lim_{x\to 2} f(x) = 0.25$$

Informal Description of the Limit

If *f(x)* becomes arbitrarily close to a single number *L* as *x* approaches *c* from either side, the limit of *f(x)*, as *x* approaches *c*, is *L*.

$$\lim_{x \to c} f(x) = L$$

Note: the existence or nonexistence of *f(x)* at *x=c* has no bearing on the existence of the limit as *x* approaches *c*.

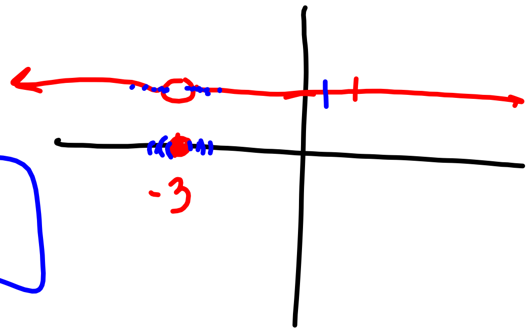
A function can be undefined for a certain value of *c* with the limit as *x* approaches *c* still defined.

$$\lim_{x \to -3} \frac{\sqrt{1-x} - 2}{x+3} = -0.25$$

$$f(x) = \begin{cases} 1, & x \neq -3 \\ 0, & x = -3 \end{cases}$$

$y = 1$

$f(-3) = 0$



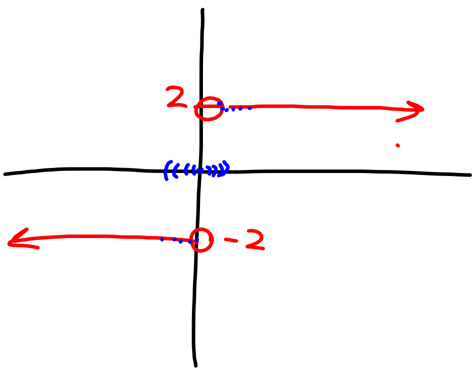
$$\lim_{x \to -3} f(x) = \boxed{1}$$

$$\lim_{x \to 0} \frac{|2x|}{x}$$

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$|-2| = -(-2) = 2$

$$\frac{|2x|}{x} = \begin{cases} \frac{2x}{x} = 2, & 2x > 0 \\ & x > 0 \\ \frac{-(2x)}{x} = -2, & 2x < 0 \\ & x < 0 \end{cases}$$



$\lim_{x \to 0} f(x) = ?$

does not exist

$\lim_{x \to 0^+} f(x) = 2$

$\lim_{x \to 0^-} f(x) = -2$

Homework:
1.2 #1-7odd,9-18all

due Friday