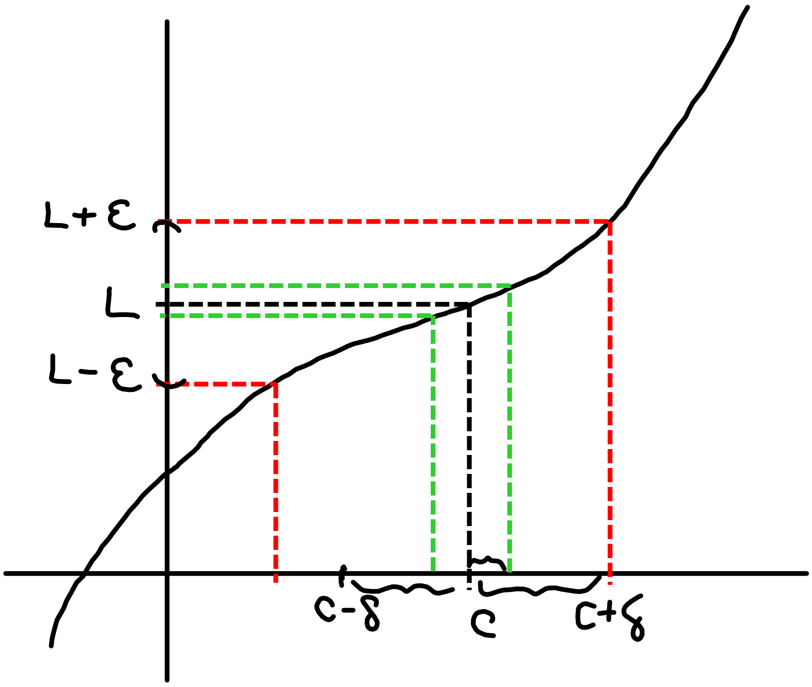
$\varepsilon - \delta$ Definition of the Limit:

$\lim_{x \to c} f(x) = L$ if given $\varepsilon > 0$, there exists a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.



$\varepsilon - \delta$ Definition of the Limit:

$\lim_{x \to c} f(x) = L$ if given $\varepsilon > 0$, there exists a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.

$L = 7$; $c = 4$

$f(x) = 2x - 1$

Find $\lim_{x \to 4} f(x)$ and prove that is the limit using the ε-δ definition.

$\lim_{x \to 4} (2x - 1) = 7$

Proof: Let $\varepsilon > 0$ be given. We want to find $\delta > 0$ such that whenever $0 < |x - c| < \delta$, we have $|f(x) - L| < \varepsilon$.

$|f(x) - L| = |2x - 1 - 7| = |2x - 8| = |2(x-4)| = 2|x-4| < \varepsilon$

$\Leftrightarrow |x - 4| < \varepsilon/2$. Take $\boxed{\delta = \varepsilon/2}$.

So whenever $|x - c| = |x - 4| < \delta = \varepsilon/2$,

we have $|f(x) - L| = 2|x - 4| < 2 \cdot \delta = 2 \cdot \varepsilon/2 = \varepsilon$.

Hence $\lim_{x \to 4} (2x - 1)$ is indeed 7.

$\varepsilon - \delta$ Definition of the Limit:

$\lim_{x\to c} f(x) = L$ if given $\varepsilon > 0$, there exists a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.

$f(x) = -5x+3$; find $\lim_{x\to 1} f(x)$ & find a δ.

$\lim_{x\to 1}(-5x+3) = -5(1)+3 = -2$

$\boxed{L=-2}$; $c=1$

$|f(x)-L| = |-5x+3-(-2)| = |-5x+5| = |-5(x-1)| =$

$|-5||x-1| = 5|x-1| < \varepsilon \iff |x-1| < \varepsilon/5$

Take $\boxed{\delta = \varepsilon/5}$

Prove that the limit is L using the $\varepsilon - \delta$ definition of the limit.

28. $\lim_{x\to -3}(2x+5) = 2(-3)+5 = -6+5 = -1 = L$

$c=-3$

Proof: Let $\varepsilon>0$ be given. To find $\delta>0$ such that $|f(x)-L|<\varepsilon$ whenever $|x-c|<\delta$.

$|f(x)-L| = |2x+5-(-1)| = |2x+6| = 2|x+3| = 2|x-(-3)| < \varepsilon$

$\iff |x-(-3)| < \varepsilon/2$.

Take $\delta = \varepsilon/2$.

Then whenever $|x-c| = |x-(-3)| < \delta = \varepsilon/2$, we have that

$|f(x)-L| = 2|x-(-3)| < 2\cdot\delta = 2\cdot\varepsilon/2 = \varepsilon$, i.e. $|f(x)-L|<\varepsilon$.

Hence, $\lim_{x\to -3}(2x+5)$ is indeed -1.

Find δ for $\varepsilon = 0.01$

24. $\lim_{x\to 4}\left(4-\frac{x}{2}\right) = 4 - \frac{4}{2} = 4-2 = 2 = L$

$c=4$

$|f(x)-L| = \left|4-\frac{x}{2}-2\right| = \left|-\frac{x}{2}+2\right| = \left|-\frac{1}{2}(x-4)\right| =$

$\frac{1}{2}|x-4| < 0.01$

$|x-4| < \boxed{0.02 = \delta}$

Significance: As long as $\underset{c-\delta}{3.98} < x < \underset{c+\delta}{4.02}$, then

$\underset{L-\varepsilon}{1.99} < f(x) < \underset{L+\varepsilon}{2.01}$.

Find δ for $\varepsilon = 0.01$

26. $\lim_{x\to 5}(x^2+4) = 5^2+4 = 25+4 = 29 = L$

$c=5$

$|f(x)-L| = |x^2+4-29| = |x^2-25| =$

$= |(x+5)(x-5)| < 11|x-5| < \varepsilon = 0.01$

Really close to 5, x is less than 6.

If $x<6$, $x+5<6+5=11$

Take $\delta = \varepsilon/11 = \boxed{\frac{0.01}{11}}$

Wherever $|x-5| < \frac{0.01}{11}$, $|f(x)-L| = |(x+5)(x-5)| < 11|x-5| <$

$< 11\cdot\frac{0.01}{11} = 0.01$; i.e. $|f(x)-L| < 0.01$.

Basic Limits

$a, c \in \mathbb{R}$

$n \in \mathbb{N}$

$$\lim_{x \to c} a = a$$

$$\lim_{x \to 5} (-3) = -3$$

$$\lim_{x \to c} x = c$$

$$\lim_{x \to -\pi} x = -\pi$$

$$\lim_{x \to c} x^n = c^n$$

$$\lim_{x \to -1} x^5 = (-1)^5 = -1$$

Theorem 1.2 more properties of Limits

$b, c \in \mathbb{R}$, $n > 0$ an integer, f & g - functions

$\lim_{x \to c} f(x) = L$; $\lim_{x \to c} g(x) = K$

1. scalar multiple

$$\lim_{x \to c} [b f(x)] = bL$$

2. sum or difference

$$\lim_{x \to c} [f(x) \pm g(x)] = L \pm K$$

3. product

$$\lim_{x \to c} [f(x) g(x)] = LK$$

4. quotient

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{K}, \quad K \neq 0$$

5. power

$$\lim_{x \to c} [f(x)]^n = L^n$$ (follows from #3)

polynomials, rational functions, $\sqrt[n]{x}$, $f(g(x))$, sin, cos, etc.

1.3

12. $\lim\limits_{x \to 1} \left(3x^3 - 2x^2 + 4\right) = 3 - 2 + 4 = \boxed{5}$

18. $\lim\limits_{x \to 3} \dfrac{\sqrt{x+1}}{x-4} = \dfrac{\sqrt{3+1}}{3-4} = \dfrac{\sqrt{4}}{-1} = \boxed{-2}$

HW #1 (submitted 11/7):
1.2 #1-7odd,9-18all

HW #2 (due 11/14):

- **1.2 #23, 25, 27, 29, 30, 31**
  and watch all of the Khan Academy epsilon-delta videos!
- **1.3 #11,17,27-35odd, 39**