HW #1 (submitted 11/7):
- 1.2 #1-7odd,9-18all

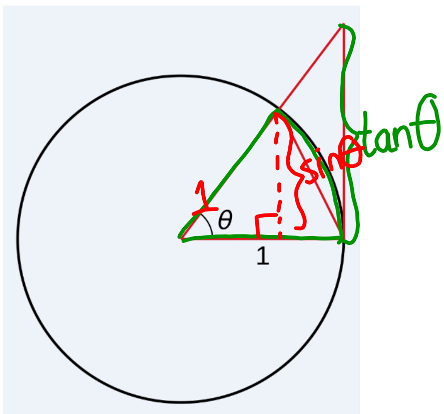
HW #2 (due 11/14):
- **1.2 #23, 25, 27, 29, 30, 31** **epsilon delta**
  (and watch all of the Khan Academy epsilon-delta videos!)
- **1.3 #11,17,27-35odd, 39-61odd** **evaluating limits analytically**
  1.3 #67-77odd; 87, 88 **limits with trig, squeeze theorem**
  ***problems in red are NOT listed on syllabus**
- **1.4 #7-17odd;** **limits of functions with discontinuities**
  1.4 #25-28all; 39-47odd; **discuss (dis)continuity**

HW #3
- 1.4 #19,21,23,51,57, 59,63,69,71 misc. continuity problems
  1.4 #83,85 intermediate value theorem

- When to have 1st quiz? — Mon 11/17
- When to have 1st test? — Wed 11/19
  Note that Fri, 11/21 is a short day, then we have a week off.

1.3 The Squeeze Theorem

$$\lim_{x\to 0}\frac{\sin x}{x} = ?$$



Area of whole circle $= \pi r^2|_{r=1} = \pi$

$$\frac{\text{Area of whole circle}}{\text{Total angle of circle}} = \frac{\text{Area of sector}}{\theta}$$

$$\frac{\pi}{2\pi} = \frac{\text{Area of sector}}{\theta} \rightarrow \text{Area of sector} = \frac{\theta}{2}$$

Area of outer triangle $\geq$ Area of sector $\geq$ Area of inner triangle

$$\frac{\tan\theta}{2} \geq \frac{\theta}{2} \geq \frac{\sin\theta}{2}$$

Multiply through by $\frac{2}{\sin\theta}$

$$\frac{\sin\theta}{2\cos\theta}\cdot\frac{2}{\sin\theta} \geq \frac{\theta}{2}\cdot\frac{2}{\sin\theta} \geq \frac{\sin\theta}{2}\cdot\frac{2}{\sin\theta}$$

$$\frac{1}{\cos\theta} \geq \frac{\theta}{\sin\theta} \geq 1$$

Take reciprocals and reverse inequalities

$$\cos\theta \leq \frac{\sin\theta}{\theta} \leq 1$$

Take limits

$$\lim_{\theta\to 0}\cos\theta \quad \le \quad \lim_{\theta\to 0}\frac{\sin\theta}{\theta} \quad \le \quad \lim_{\theta\to 0} 1$$

$$1 \le \lim_{\theta\to 0}\frac{\sin\theta}{\theta} \le 1$$

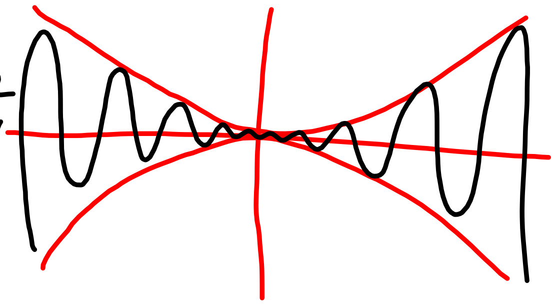
$$\Rightarrow \boxed{\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1}$$

The Squeeze Theorem:

If $f(x) \le g(x) \le h(x)$ and $\lim_{x\to c} f(x) = L = \lim_{x\to c} h(x)$,

Then $\lim_{x\to c} g(x) = L$.

$$-x^2 \le x^2\sin x \le x^2$$



Special Limits Derived by Squeeze Theorem:

$$\boxed{\lim_{x\to 0}\frac{\sin x}{x} = 1} \; ; \; \boxed{\lim_{x\to 0}\frac{1-\cos x}{x} = 0}$$

Memorize!!

Use the squeeze theorem to find

$$\lim_{x\to 0}\left(x^2\cos\frac{5}{x}-3\right)$$

$$-1 \le \cos\theta \le 1$$

$$-1 \le \cos\frac{5}{x} \le 1$$

$$-x^2 \le x^2\cos\frac{5}{x} \le x^2$$

$$-x^2-3 \le x^2\cos\frac{5}{x}-3 \le x^2-3$$

$$\lim_{x\to 0}(-x^2-3) \le \lim_{x\to 0}\left(x^2\cos\frac{5}{x}-3\right) \le \lim_{x\to 0}(x^2-3)$$

$$-3 \le \lim_{x\to 0}\left(x^2\cos\frac{5}{x}-3\right) \le -3$$

By the Squeeze Theorem,

$$\lim_{x\to 0}\left(x^2\cos\frac{5}{x}-3\right) = \boxed{-3}$$

68. $\displaystyle\lim_{x\to 0}\frac{3(1-\cos x)}{x}$

$$= 3\cdot\lim_{x\to 0}\frac{(1-\cos x)}{x}$$

$$= 3\cdot 0$$

$$= \boxed{0}$$

72. $\lim_{x\to 0} \frac{\tan^2 x}{x} = \lim_{x\to 0} \frac{\sin^2 x}{\cos^2 x \cdot x}$

$= \lim_{x\to 0} \left[\left(\frac{\sin x}{x}\right)\left(\frac{\sin x}{\cos^2 x}\right) \right]$

$= \left[\lim_{x\to 0} \frac{\sin x}{x} \right] \cdot \left[\lim_{x\to 0} \frac{\sin x}{\cos^2 x} \right]$

$= 1 \cdot \frac{\sin 0}{\cos^2 0}$

$= 1 \cdot \frac{0}{1} = 1 \cdot 0 = \boxed{0}$

78. $\lim_{x\to 0} \frac{\sin 2x}{\sin 3x}$

$\lim_{x\to 0} \frac{\sin x}{x} = 1$

$\lim_{2x\to 0} \frac{\sin(2x)}{2x} = 1$

$= \lim_{x\to 0} \frac{\sin 2x}{2x} \cdot \frac{3x}{\sin 3x} \cdot \frac{2}{3}$

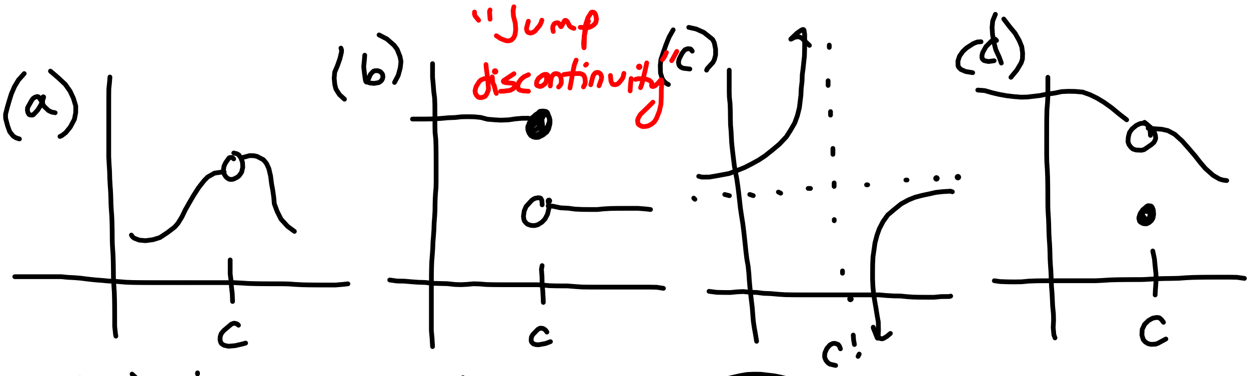
$= \lim_{x\to 0} \frac{\sin 2x}{2x} \cdot \frac{1}{\frac{\sin 3x}{3x}} \cdot \frac{2}{3}$

$= \left(\lim_{x\to 0} \frac{\sin 2x}{2x}\right)\left(\lim_{x\to 0} \frac{1}{\frac{\sin 3x}{3x}}\right)\left(\lim_{x\to 0} \frac{2}{3}\right)$

$= 1 \cdot \left(\frac{\lim_{x\to 0} 1}{\lim_{x\to 0} \frac{\sin 3x}{3x}}\right) \cdot \frac{2}{3}$

$= 1 \cdot \frac{1}{1} \cdot \frac{2}{3} = \boxed{2/3}$

1.4 Continuity and One-Sided Limits

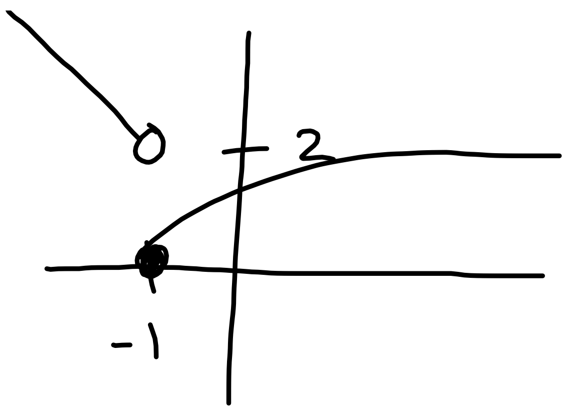


f(c) is undefined

$\lim_{x\to c} f(x)$ does not exist

$\lim_{x\to c} f(x) \neq f(c)$

These are all discontinuities
(a) and (d) are removable
(b) and (c) are nonremovable



$$\lim_{x\to -1^-} f(x) = 2$$

$$\lim_{x\to -1^+} f(x) = 0$$

$$\lim_{x\to -1} f(x) = \text{does not exist}$$

One-Sided Limits

$\lim_{x\to c^+} f(x) = L$ *limit from the right*

$\lim_{x\to c^-} f(x) = L$ *limit from the left*

$$\lim_{x\to c} f(x) = L \textit{ if and only if}$$

$$\lim_{x\to c^-} f(x) = L = \lim_{x\to c^+} f(x)$$

Continuity at a point

A function f is continuous at c if the following 3 conditions are met:

1. $f(c)$ is defined
2. Limit of $f(x)$ exists when x approaches c
3. Limit of $f(x)$ when x approaches c is equal to $f(c)$

f(x) is continuous at c if

$$\lim_{x\to c} f(x) = f(c)$$

Continuity on an open interval

A function is continuous on an open interval if it is continuous at each point in the interval. A function that is continuous on the entire real line $(-\infty, \infty)$is everywhere continuous.

Continuity on a closed interval

A function f is continuous on the closed interval $[a, b]$ if it is continuous on the open interval I (a, b) and $\lim_{x\to a^+} f(x) = f(a)$ and $\lim_{x\to b^-} f(x) = f(b)$.

10. $$\lim_{x\to 4^-} \frac{\sqrt{x}-2}{x-4} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2}$$

$$= \lim_{x\to 4^-} \frac{\cancel{x-4}\ 1}{\cancel{(x-4)}(\sqrt{x}+2)}$$

$$= \lim_{x\to 4^-} \frac{1}{\sqrt{4}+2} = \boxed{\frac{1}{4}}$$

12. $\lim_{x \to 2^+} \frac{|x-2|}{x-2} = \boxed{1}$

$$\frac{|x-2|}{x-2} = \begin{cases} \frac{x-2}{x-2} = 1, & x-2>0 \\ & x>2 \\ \frac{-(x-2)}{x-2} = -1, & x-2<0 \\ & x<2 \end{cases}$$

1.4

Discuss the [dis]continuity of the function.

$$f(x) = \frac{(x+4)(x-2)}{(x-2)(x+1)}$$

removable discontinuity @ $x=2$

non-removable discontinuity @ $x=-1$

f is continuous on : $(-\infty,-1)\cup(-1,2)\cup(2,\infty)$

$$f(x) = \frac{|x-2|}{x-2}$$

non-removable (jump) discontinuity @ $x=2$

continuous on $(-\infty, 2) \cup (2, \infty)$

$$f(x) = \begin{cases} x^2-2, & x \geq 1 \\ 5, & x < 1 \end{cases}$$

non-removable (jump) discontinuity @ $x=1$

$(-\infty, 1) \cup [1, \infty)$