Homework for Test #1:

HW #1 (submitted 11/7):
- 1.2 #1-7odd,9-18all

HW #2 (submitted 11/14):
- 1.2 #23, 25, 27, 29, 30, 31 — epsilon delta
- 1.3 #11,17,27-35odd, 39-61odd — evaluating limits analytically
  1.3 #67-77odd; 87, 88 — limits with trig, squeeze theorem
- 1.4 #7-17odd; — limits of functions with discontinuities
  1.4 #25-28all; 39-47odd; — discuss (dis)continuity

HW #3 (due Fri. 11/21)
- 1.4 #19,21,23,51,57,59,63,69,71 — misc. continuity problems
  1.4 #83,85 — intermediate value theorem
- 1.5 #1,3,25; 29-51odd — infinite limits
- Ch 1 review pp. 88-89 #3-49odd; 51-67odd
- Test #1 Practice Problems

Test #1 - Friday, 11/21

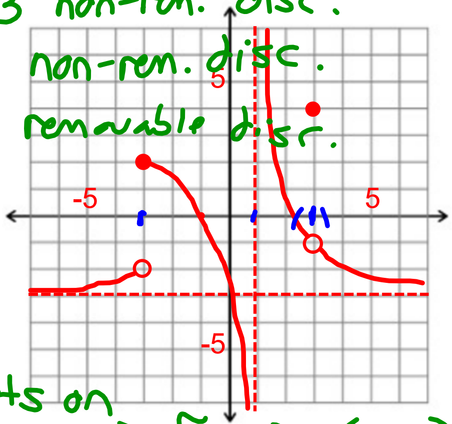
Discuss the continuity of the following function. State which discontinuities are removable and which are non-removable, and state the intervals on which the function is continuous.

1. $f(x) = \frac{x-6}{x^2-7x+6}$ $= \frac{x-6}{(x-6)(x-1)}$

removable discontinuity @ $x=6$
non-removable discontinuity @ $x=1$
continuous on $(-\infty,1)\cup(1,6)\cup(6,\infty)$

Assume that the graph to the left is $f(x)$. Find the requested limits.

-3 non-rem. disc.
1 non-rem. disc.
3 removable disc.



cts on
$(-\infty,-3)\cup[-3,1)\cup(1,3)\cup(3,\infty)$

2. $\lim_{x\to 3} f(x)$ -1

3. $\lim_{x\to -3^-} f(x)$ -2

4. $\lim_{x\to 1^+} f(x)$ ∞

5. $\lim_{x\to -3^+} f(x)$ 2

6. $\lim_{x\to 1^-} f(x)$ $-\infty$

7. A function f is continuous at c if $\lim_{x\to c} f(x) =$ $f(c)$.

8. According to the Squeeze Theorem if $f(x) \le g(x) \le h(x)$, and $\lim_{x\to c} f(x) = L = \lim_{x\to c} h(x)$, then $\lim_{x\to c} g(x) =$ L.

9. State the epsilon-delta definition of the statement $\lim_{x\to c} f(x) = L$.

Given $\varepsilon > 0$, there exists a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $|x - c| < \delta$

($|x-c| < \delta$ implies that $|f(x) - L| < \varepsilon$)

Find the limits (if they exist). Draw a picture if this helps. Circle/box your final answers.

10. $\lim_{x\to 3} \frac{|x-3|}{3-x}$

$$\frac{|x-3|}{3-x} = \begin{cases} \frac{x-3}{3-x} = -1, & x-3>0 \\ & x>3 \\ \frac{-(x-3)}{3-x} = 1, & x-3<0 \\ & x<3 \end{cases}$$

$\boxed{\text{does not exist}}$

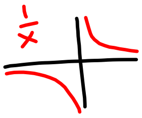
11. $\lim_{x\to 5} \frac{\sqrt{x-1}}{x-4} = \frac{\sqrt{5-1}}{5-4} = \frac{\sqrt{4}}{1} = \boxed{2}$

12. $\lim_{x\to 0} \frac{\sin x}{x} = \boxed{1}$

13. $\lim_{x\to 0} \frac{1-\cos x}{x} = \boxed{0}$

42. $\lim\limits_{x\to 0^-}\left(x^2-\frac{1}{x}\right) = \lim\limits_{x\to 0^-}(x^2) - \lim\limits_{x\to 0^-}\left(\frac{1}{x}\right)$

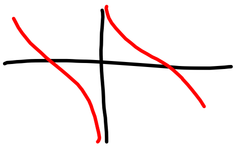
$\frac{1}{x}$



$= 0-(-\infty)$

$= \boxed{\infty}$

46. $\lim\limits_{x\to 0}\frac{x+2}{\cot x} = \frac{\lim\limits_{x\to 0}(x+2)}{\lim\limits_{x\to 0}(\cot x)} = \frac{2}{\pm\infty}$



$\to \boxed{0}$

$= \lim\limits_{x\to 0}(x+2)\cdot\tan x$

$= \left(\lim\limits_{x\to 0}(x+2)\right)\left(\lim\limits_{x\to 0}\frac{\sin x}{\cos x}\right)$

$= 2\cdot\frac{0}{1} = \boxed{0}$

8. Determine if the Intermediate Value Theorem guarantees a c in the interval $[-2,3]$ such that $f(c)=-4$, and if so, find all such values of c.

$f(x)=x^2-7x+2$

$f(x)$ continuous on $[-2,3]$? yes

$f(-2)=(-2)^2-7(-2)+2 = 4+14+2=20 > -4$

$f(3)=(3)^2-7(3)+2 = 9-21+2=-10 < -4$

IVT does apply

$f(c)=-4$

$c^2-7c+2=-4$

$c^2-7c+6=0$

$(c-6)(c-1)=0$

$\cancel{c=6}$; $\boxed{c=1}$

Find the limit (if it exists).

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}, \quad where\ f(x)=3x^2+1$$

$$\lim_{\Delta x \to 0} \frac{3(x+\Delta x)^2+1-(3x^2+1)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{3(x^2+2x\cdot\Delta x+(\Delta x)^2)+1-3x^2-1}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{3x^2}+6x\cdot\Delta x+3(\Delta x)^2\cancel{+1}\cancel{-3x^2}\cancel{-1}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(6x+3\cdot\Delta x)}{\cancel{\Delta x}} = 6x+3(0) = \boxed{6x}$$

Find the limit (if it exists).

$$\lim_{x \to 2} \frac{(\sqrt{x+7}-3)}{(x^2-x-2)} \cdot \frac{(\sqrt{x+7}+3)}{(\sqrt{x+7}+3)}$$

$$= \lim_{x \to 2} \frac{x+7-9}{(x^2-x-2)(\sqrt{x+7}+3)}$$

$$= \lim_{x \to 2} \frac{\cancel{x-2}\ 1}{\cancel{(x-2)}(x+1)(\sqrt{x+7}+3)}$$

$$= \frac{1}{(2+1)(\sqrt{2+7}+3)} = \frac{1}{3(\sqrt{9}+3)} = \frac{1}{3(3+3)}$$

$$= \frac{1}{3\cdot 6} = \boxed{\frac{1}{18}}$$

9. Discuss the continuity of the function (identify all discontinuities, if any, as removable or non-removable).

$$f(x) = \frac{x^2 - 7x + 10}{x^2 - 3x + 2} = \frac{(x-5)\cancel{(x-2)}}{(x-1)\cancel{(x-2)}} = \frac{x-5}{x-1}$$

non-removable discontinuity @ $x = 1$

removable discontinuity @ $x = 2$

$f(x)$ is continuous on $(-\infty, 1) \cup (1, 2) \cup (2, \infty)$

5. Find the limit (if it exists).

$$\lim_{x \to -5} f(x), \quad f(x) = \begin{cases} -x^2 + 8, & x \leq -5 \\ 2x + 3, & x > -5 \end{cases}$$

$$\lim_{x \to -5^-} f(x) = -(-5)^2 + 8 = -25 + 8 = -17$$

$$\lim_{x \to -5^+} f(x) = 2(-5) + 3 = -10 + 3 = -7$$

limit does not exist

Use the Squeeze Theorem to find $\lim_{x\to 0} f(x)$. $= \boxed{0}$

$$f(x) = x^2 \cos\frac{1}{x}$$

$-1 \le \cos\theta \le 1$

$$-1 \le \cos\frac{1}{x} \le 1$$

$$-x^2 \le x^2\cos\frac{1}{x} \le x^2$$

$$\lim_{x\to 0}(-x^2) \le \lim_{x\to 0} x^2\cos\frac{1}{x} \le \lim_{x\to 0} x^2$$

$$0 \le \lim_{x\to 0} x^2\cos\frac{1}{x} \le 0$$

Find the limit, then use the $\varepsilon - \delta$ definition to prove that the limit is L .

$$\lim_{x\to 4}(3x + 2)$$

Find the limit (if it exists).

$$\lim_{x \to 0} \frac{\sin 2x}{\sin 3x}$$

Find the limit (if it exists).

$$\lim_{x \to 3^-} \frac{-7|x-3|}{3-x}$$

Find the limit (if it exists).

$$\lim_{x \to \frac{\pi}{2}^+} \frac{3}{\tan x}$$

Find the vertical asymptotes (if any).

14. $f(x) = \dfrac{-4x}{x^2+4}$

24. $h(x) = \dfrac{x^2-4}{x^3+2x^2+x+2}$

28. $g(\theta) = \dfrac{\tan\theta}{\theta}$