What happens if . . .

$$x^2y + y^2x = -2$$

how to find y'?

2.5 Implicit Differentiation

☆ $y = f(x)$

y is a function of x

$$\frac{d}{dx}[x] = 1 \quad ; \quad \frac{d}{dx}[y] = y'$$

$$\frac{d}{dx}[\sin x] = \cos x \quad ; \quad \frac{d}{dx}[\sin y] = (\cos y) \cdot y'$$

6. $x^2y + y^2x = -2$

$$\frac{d}{dx}\left[x^2y + y^2x\right] = \frac{d}{dx}\left[-2\right]$$

$$(x^2)'y + (x^2)(y)' + (y^2)'x + (y^2)(x)' = 0$$

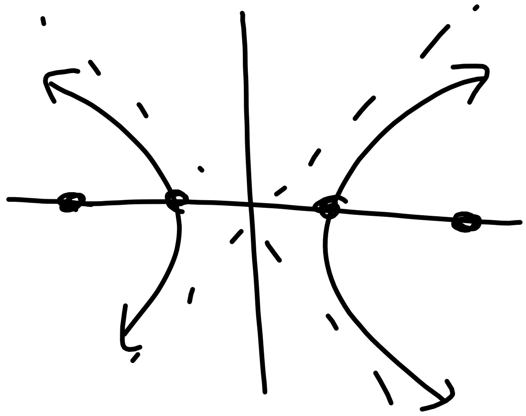
$$2xy + x^2y' + 2y \cdot y'x + y^2 \cdot 1 = 0$$

$$x^2y' + 2xyy' = -y^2 - 2xy$$

$$y'(x^2 + 2xy) = -y^2 - 2xy$$

$$\boxed{y' = \frac{-y^2 - 2xy}{x^2 + 2xy}}$$

2. $x^2 - y^2 = 16$



$$\frac{d}{dx}\left[x^2 - y^2\right] = \frac{d}{dx}\left[16\right]$$

$$2x - 2yy' = 0$$

$$2x = 2yy'$$

$$\frac{2x}{2y} = y'$$

$$\boxed{\frac{x}{y} = y'}$$

8. $\sqrt{xy} = x - 2y$ $\quad (xy)^{1/2} = x - 2y$

$$\frac{d}{dx}\left[(xy)^{1/2}\right] = \frac{d}{dx}\left[x - 2y\right]$$

$$\frac{1}{2}(xy)^{-1/2} \cdot \left[xy' + x'y\right] = \left[x' - 2y'\right]$$

$$\frac{xy' + y}{2\sqrt{xy}} = 1 - 2y'$$

$$xy' + y = 2\sqrt{xy} - 4y'\sqrt{xy}$$

$$xy' + 4y'\sqrt{xy} = 2\sqrt{xy} - y$$

$$y'(x + 4\sqrt{xy}) = 2\sqrt{xy} - y$$

$$\boxed{y' = \frac{2\sqrt{xy} - y}{x + 4\sqrt{xy}}}$$

10. $2\sin x\cos y = 1$

$$\frac{d}{dx}\left[2\sin x\cos y\right] = \frac{d}{dx}\left[1\right]$$

$$(2\sin x)' \cdot \cos y + (2\sin x)(\cos y)' = 0$$

$$2\cos x\cos y + 2\sin x\,(-\sin y \cdot y') = 0$$

$$2\cos x\cos y = 2y'\sin x\sin y$$

$$\frac{2\cos x\cos y}{2\sin x\sin y} = y'$$

$$\boxed{\cot x\cot y = y'}$$

12. $(\sin \pi x + \cos \pi y)^2 = 2$

$$\frac{d}{dx}\left[(\sin \pi x + \cos \pi y)^2\right] = \frac{d}{dx}[2]$$

$$\underbrace{2(\sin \pi x + \cos \pi y)} \cdot (\cos \pi x \cdot \pi - \sin \pi y \cdot (\pi y')) = 0$$

$$2\pi \sin \pi x \cos \pi x + 2\pi \cos \pi x \cos \pi y = 2\pi y' \sin \pi y (\sin \pi x + \cos \pi y)$$

$$\frac{2\pi \sin \pi x \cos \pi x + 2\pi \cos \pi x \cos \pi y}{2\pi \sin \pi y(\sin \pi x + \cos \pi y)} = y'$$

$$\frac{\cancel{2\pi} \cos \pi x \,(\cancel{\sin \pi x + \cos \pi y})}{\cancel{2\pi} \sin \pi y\,(\cancel{\sin \pi x + \cos \pi y})} = y'$$

$$\boxed{\frac{\cos \pi x}{\sin \pi y} = y'}$$

16. $x = \sec \frac{1}{y}$

$\frac{1}{y} = y^{-1}$

$(y^{-1})' = -1 \cdot y^{-1-1}$

$$\frac{d}{dx}[x] = \frac{d}{dx}\left[\sec \frac{1}{y}\right]$$

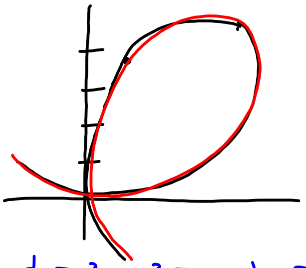
$$1 = \sec \frac{1}{y} \tan \frac{1}{y} \cdot \left(\frac{1}{y}\right)'$$

$$1 = \sec \frac{1}{y} \tan \frac{1}{y} \cdot (-y^{-2} \cdot y')$$

$$\frac{1}{\sec \frac{1}{y} \tan \frac{1}{y} (-y^{-2})} = y'$$

$$\boxed{-y^2 \cos \frac{1}{y} \cot \frac{1}{y} = y'}$$

32. Folium of Descartes



$x^3 + y^3 - 6xy = 0$

find the slope of the tangent line @ $\left(\frac{4}{3}, \frac{8}{3}\right)$

$$\frac{d}{dx}\left[x^3 + y^3\right] = \frac{d}{dx}\left[6xy\right]$$

$$3x^2 + 3y^2 y' = (6)y + (6x)y'$$

$$y'(3y^2 - 6x) = 6y - 3x^2$$

$$y' = \frac{3(2y - x^2)}{3(y^2 - 2x)} = \frac{2y - x^2}{y^2 - 2x}$$

$$m = \frac{2\left(\frac{8}{3}\right) - \left(\frac{4}{3}\right)^2}{\left(\frac{8}{3}\right)^2 - 2\left(\frac{4}{3}\right)} = \frac{\frac{16}{3} - \frac{16}{9}}{\frac{64}{9} - \frac{8}{3}} = \frac{\frac{48-16}{9}}{\frac{64-24}{9}}$$

$$= \frac{32}{40} = \boxed{\frac{4}{5}}$$

HW #7 (due Fri, 9 Jan)
2.5 # 1-39 odd; 43, 47 - Implicit Differentiation
2.6 # 15-23 odd - Related Rates
2.6 # 25, 27, 35 - Related Rates (more challenging problems)

Quiz #4 - Fri, 9 Jan?

HW #8 (due Fri, 16 Jan)
3.1 # 17-31 odd - Absolute Extrema on an Interval
3.2 # 7-19 odd - Rolle's Theorem
3.2 # 31-37 odd - Mean Value Theorem
3.3 # 11-31 odd - Increasing, Decreasing, and Relative Extrema

Quiz #5 - Fri, 16 Jan?

HW #9 (due Test Day)
3.4 #11-25 odd - Inflection Points and Concavity
3.5 #15-31 odd - Limits at Infinity

Test #3 - Wed, 21 Jan?