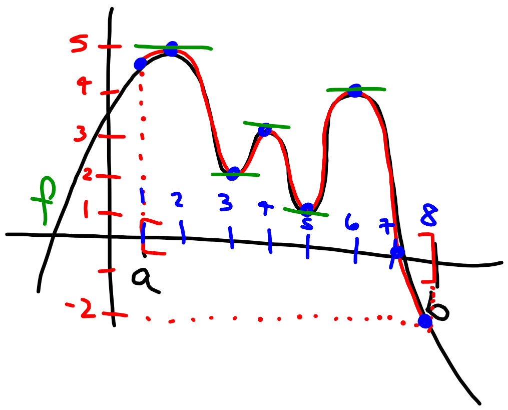
3.1 Extrema on an Interval

↳ maxima & minima

↳ relative & absolute



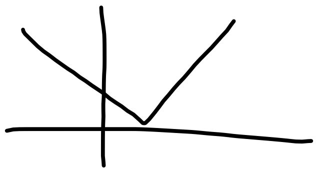
relative minima:
(3,2), (5,1)

relative maxima:
(2,5), (4,3), (6,4)

absolute maximum:
5 @ (2,5)

absolute minimum:
−2 @ (8,−2)

f(x) has a relative maximum or minimum when $f'(x) = 0$. or $f'(x)$ is undefined.



We call such x-values **Critical #'s of f**.

3.1

28. $h(t)=\dfrac{t}{t-2}$, $[3,5]$

Find the absolute max & min on the closed interval.

step 1: take derivative & find critical #'s

$$h'(t)=\frac{(t-2)(1)-t(1)}{(t-2)^2}=\frac{-2}{(t-2)^2}$$

critical #: 2 (not in $[3,5]$)

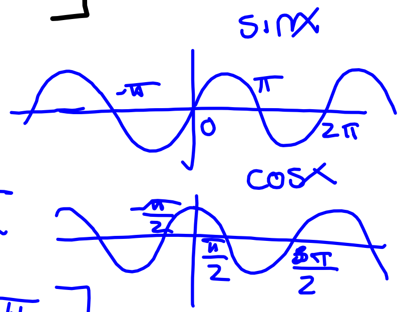
Step 2: plug critical #'s & endpoints into original function

$h(3)=\dfrac{3}{3-2}=3$ ← absolute max

$h(5)=\dfrac{5}{5-2}=5/3$ ← absolute min

30. $g(x)=\sec x$, $\left[-\frac{\pi}{6},\frac{\pi}{3}\right]$

Find the absolute max & min on the closed interval.



$g'(x)=\sec x\tan x=\dfrac{1}{\cos x}\cdot\dfrac{\sin x}{\cos x}$

$g'(x)=0$ @ $x=0$

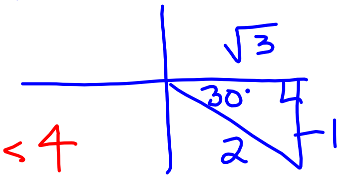
$g'(x)$ is never defined ~~@ x~~ on $[-\pi/6, \pi/3]$

Critical #: 0

$g(-\pi/6)=\sec(-\pi/6)=\dfrac{2}{\sqrt{3}}$

$g(0)=\sec(0)=1$ abs min

$g(\pi/3)=\sec(\pi/3)=2$ abs. max

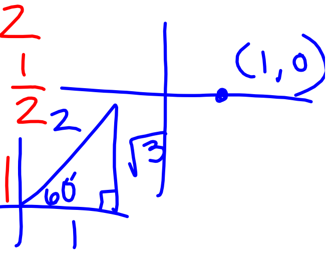


$1<3<4$

$1<\sqrt{3}<2$

$1>\dfrac{1}{\sqrt{3}}>\dfrac{1}{2}$

$2>\dfrac{2}{\sqrt{3}}>1$



22. $f(x) = x^3 - 12x$, $[0, 4]$

Find the absolute max & min on the closed interval.

$f'(x) = 3x^2 - 12$

$3x^2 - 12 = 0$

$3x^2 = 12$

$x^2 = 4$

$x = \pm 2$

critical #s: 2, -2

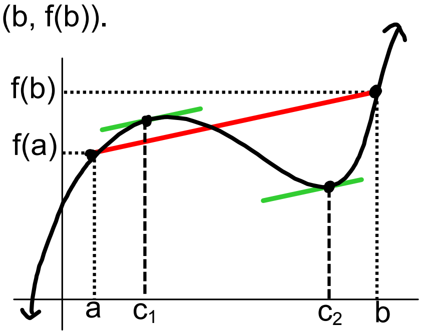
$f(0) = 0^3 - 12(0) = 0$

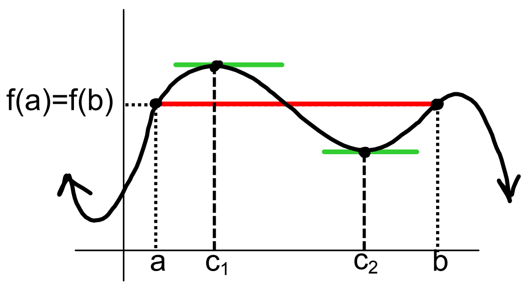
$f(2) = 2^3 - 12(2) = -16$ abs min

$f(4) = 4^3 - 12(4) = 16$ abs max

3.2 Rolle's Theorem & The Mean Value Theorem

The Mean Value Thereom (MVT) states: If f is continuous on [a,b] and differentiable on (a,b), then there exists at least one c in (a,b) such that the slope of the tangent line at c is equal to the slope of the secant line through (a, f(a)) and (b, f(b)).





Rolle's Theorem is a special case of the MVT where f(a)=f(b), (and hence involving horizontal secant/tangent lines)

Note that neither the Mean Value Theorem nor Rolle's Theorem apply to the following functions on the given intervals:

$$f(x) = \frac{x+5}{x-2}, \qquad [1,3]$$

f is not continuous on [1,3].

$$g(x) = |x-2|, \qquad [1,3]$$

g is continuous on [1,3], but not differentiable on (1,3).

Can Rolle's Theorem be applied?
If so, find all guaranteed values of c in (a,b).

8. $f(x) = x^2 - 5x + 4, \quad [1,4]$

Is f continuous on [1,4]? yes
Is f differentiable on (1,4)? yes
Is f(1) = f(4)? Yes

Rolle's Thm does apply

$f(1) = 1^2 - 5(1) + 4 = 0$

$f(4) = 4^2 - 5(4) + 4 = 0$

$f'(x) = 2x - 5$

$2x - 5 = 0$

$2x = 5$

$\boxed{x = 5/2}$

Set slope of tangent line ($f'(x)$) equal to slope of secant line $\left(\frac{f(b) - f(a)}{b - a}\right)$ & solve for x

HW #7 (due Fri, 9 Jan)
2.5 # 1-39 odd; 43, 47 - Implicit Differentiation
2.6 # 15-23 odd - Related Rates
2.6 # 25, 27, 35 - Related Rates (more challenging problems)

HW #8 (due Fri, 16 Jan)
Test 3 Practice Problems Handout (NOT ON WEB)

Quiz #5 - Fri, 16 Jan (related rates)
Test #3 - Tues, 20 Jan (implicit differentiation, related rates, & review problems)

HW #9
3.1 # 17-31 odd - Absolute Extrema on an Interval
3.2 # 7-19 odd - Rolle's Theorem
3.2 # 31-37 odd - Mean Value Theorem
3.3 # 11-31 odd - Increasing, Decreasing, and Relative Extrema
Quiz #6?
HW #10
3.4 #11-25 odd - Inflection Points and Concavity
3.5 #15-31 odd - Limits at Infinity