Differential Calculus | Quiz #5 | Name:______________________________

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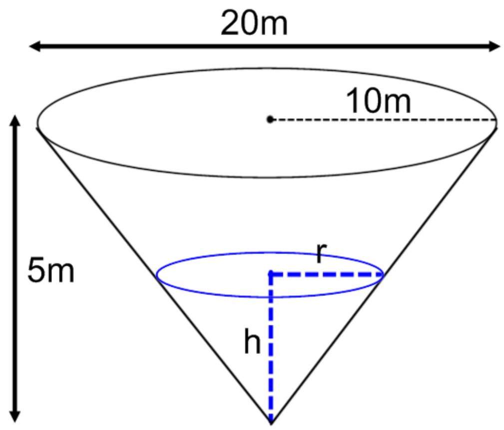
A conical tank whose top diameter is 20 meters across and total height is 5 meters is being filled at a rate of 1 cubic meter every 5 minutes. Determine the rate at which the radius of its contents is increasing when the height of the contents reaches 4 meters. Note that the volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$.

1. Draw a picture illustrating this problem, including appropriate labels for radius, height, etc.



2. State the given information with appropriate variables and units.

$$\frac{dV}{dt} = \frac{1\,m^3}{5\,min}\ ;\ h = 4m$$

3. State the desired unknown information with appropriate variables and units.

$$\frac{dr}{dt} = ?\ \frac{m}{min}$$

4. Give another formula relating some of the variables in #2 and #3 besides $V = \frac{1}{3}\pi r^2 h$.

$$\frac{h}{r} = \frac{5}{10}\ ;\ \textit{this can be rewritten as either}\ \ h = \frac{r}{2}\ \ or\ \ r = 2h,\quad \textit{as needed}$$

5. Rewrite $V = \frac{1}{3}\pi r^2 h$ utilizing the formula stated in #4 (to remove undesired variables) and simplify.

$$V = \frac{1}{3}\pi r^2 \cdot \frac{r}{2} \quad \longrightarrow \quad V = \frac{\pi}{6} r^3$$

6. Take the derivative of your simplified formula with respect to the appropriate variable.

$$\frac{dV}{dt} = \frac{\pi}{6}(3r^2) \cdot \frac{dr}{dt}$$

$$\frac{dV}{dt} = \frac{\pi}{2} r^2 \cdot \frac{dr}{dt}$$

7. Rearrange the result obtained in #6 to solve for the unknown stated in #3.

$$\frac{dr}{dt} = \frac{\frac{dV}{dt}}{\frac{\pi}{2} r^2}$$

8. Plug in any given constants to determine the exact value (as a fraction in terms of π as necessary) including appropriate units.

$$\frac{dr}{dt} = \frac{\frac{1\,m^3}{5\,min}}{\frac{\pi}{2}\frac{(2\cdot 4m)^2}{1}} = \frac{1\,m^3}{5\,min} \cdot \frac{2}{\pi \cdot 64m^2} = \boxed{\frac{1}{160\pi}\ \frac{m}{min}}$$