1.a $y' = \frac{-2xy}{x^2+1}$

b. $y' = \frac{\sec^2 x}{\cos y + \sin y}$

c. $y' = \frac{5xy - y}{x}$

2. $y = \frac{5}{3}x - \frac{16}{3}$

4. $\frac{dh}{dt} = \frac{3}{4\pi}$ ft/s

5a. $\frac{100}{3}$ vib/s/lb

b. $\frac{200}{7}$ vib/s/lb

$r = 3h$

$r = 4$ ft

$\Rightarrow h = 4/3$ ft

$V = \frac{1}{3}\pi(3h)^2 \cdot h = 3\pi h^3$

$\frac{dV}{dt} = 9\pi h^2 \cdot \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{\frac{dV}{dt}}{9\pi h^2} = \frac{12\ \text{ft}^3/\text{s}}{9\pi\left(\frac{4}{3}\text{ ft}\right)^2} = \frac{3}{4\pi}$ ft/s

3.2

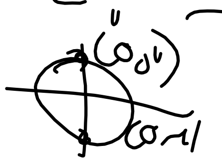
13. $f(x) = \frac{x^2-2x-3}{x+2}$, $[-1,3]$

$= \frac{(x-3)(x+1)}{x+2}$

$\cancel{4+2-3=0}$

$\cos x = 0$ $\quad [0, 2\pi]$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$



17. $f(x) = \frac{6x}{\pi} - 4\sin^2 x$ $\quad [0, \pi/6]$

$4(\sin x)^2$

$f'(x) = \frac{6}{\pi} - 8\sin x\cos x$

$\frac{6}{\pi} - 8\sin x\cos x = 0$ $\quad \frac{1}{\cancel{8}_4} \cdot \frac{\cancel{6}^3}{\pi} = \cancel{8}\sin x\cos x \cdot \frac{1}{\cancel{8}}$

$2\sin x\cos x = \frac{3}{\cancel{4}^{2}\pi}$ $\quad \sin 2x = 2\sin x\cos x$

$\sin 2x = \frac{3}{2\pi}$, $\quad 2x = \sin^{-1}\left(\frac{3}{2\pi}\right)$

$x = \frac{\sin^{-1}\left(\frac{3}{2\pi}\right)}{2} \approx \boxed{0.2489}$

$$\sin x = c$$

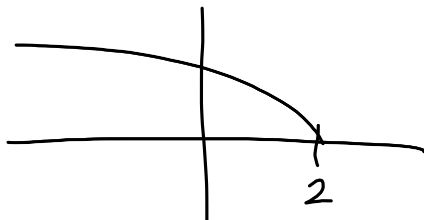
$$\sin^{-1}(\sin x) = \sin^{-1}(c)$$

$$x = \sin^{-1}(c)$$

$$(\sqrt{x})^2 = (c)^2$$

35. $f(x) = \sqrt{2-x}$, $[-7, 2]$

$$= (2-x)^{1/2}$$



$$f'(x) = \frac{1}{2}(2-x)^{-1/2}(-1)$$

$$\frac{f(b)-f(a)}{b-a} = \frac{\sqrt{0} - \sqrt{9}}{2+(+7)}$$

$$= \frac{-3}{9} = \frac{-1}{3}$$

$$\frac{-1}{3} = \frac{-1}{2\sqrt{2-x}}$$

$$2\sqrt{2-x} = 3$$

$$\sqrt{2-x} = \frac{3}{2}$$

$$2-x = \frac{9}{4}$$

$$2 - \frac{9}{4} = x$$

$$\boxed{\frac{-1}{4} = x}$$

3.3-3.4 Increasing, Decreasing, Concavity, and the 1st and 2nd Derivative Tests

What do *f'* and *f''* tell us about *f*?

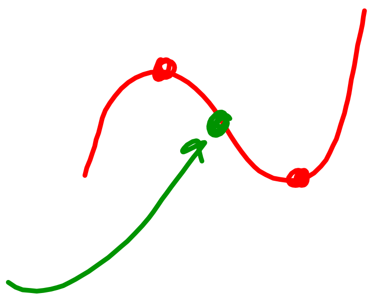
Recall that *f'* is the rate of change or slope of *f*,
f'' is the slope or rate of change of *f'*.

f'	f
+	↗ increasing
−	↘ decreasing

f''	f'	f
+	↗ increasing	concave up
−	↘ decreasing	concave down

f'(x)=0 when f has a relative maximum or minimum.
These x-values (and those where f'(x) is undefined) are called critical numbers.

f''(x)=0 when f changes concavity.
The points where concavity changes are called inflection points.



To solve problems involving concavity, increasing/decreasing, etc., we should recall how to solve polynomial inequalities.

$$\frac{(x+2)(x-3)}{x+4} \geq 0$$

-2, 3; -4

plot zeros & vertical asymptotes on number line

$-$ ⊕ -4 $+$ -2 $-$ 3 $+$

$$(-4,-2] \cup [3,\infty)$$

- Find all critical numbers and state the open intervals on which f is increasing and/or decreasing.
- Find all inflection points and state the open intervals on which f is concave up and/or concave down.
- Use these results to determine all relative and absolute extrema.

3.3

16. $f(x)=x^3-6x^2+15$

$f'(x)=3x^2-12x$

$3x(x-4)=0$

$3x=0$ $\quad x-4=0$

$x=0$ $\quad x=4$

$\boxed{\text{critical \#'s: } 0, 4}$

$f'(-1)$, $f'(1)$, $f'(5)$

$+$ 0 $-$ 4 $+$

f is increasing on $(-\infty,0)\cup(4,\infty)$
f is decreasing on $(0,4)$

f has a relative max @ $(0,15)$
& a relative min @ $(4,-17)$

$f''(x)=6x-12$

$6(x-2)=0$

$x=2$

$f''(1)$, $f''(3)$

$-$ 2 $+$

f has an inflection point @ $(2,-1)$
f is concave down on $(-\infty,2)$
& concave up on $(2,\infty)$

HW #9 (due Fri, 23 Jan)
3.1 # 17-31 odd - Absolute Extrema on an Interval
3.2 # 7-19 odd - Rolle's Theorem
3.2 # 31-37 odd - Mean Value Theorem
Quiz #6 - Mon, 26 Jan

HW #10 (due Fri, 30 Jan)
3.3 # 11-31 odd - Increasing, Decreasing, and Relative Extrema
3.4 #11-25 odd - Inflection Points and Concavity
3.5 #15-31 odd - Limits at Infinity
7.7 #11-35 odd - l'Hopital's Rule
Quiz #7 - Fri, 30 Jan

HW #11 (due Wed, 4 Feb)
7.7 #37-53 odd - l'Hopital's Rule with logs
3.7 #3,5,17,23,29 - Optimization
Test 4 - Wed, 4 Feb

Final Exam - Thurs, 12 Feb 1:00-3:00pm