HW #9 (due Fri, 23 Jan)
3.1 # 17-31 odd - Absolute Extrema on an Interval
3.2 # 7-19 odd - Rolle's Theorem
3.2 # 31-37 odd - Mean Value Theorem
Quiz #6 - Mon, 26 Jan

HW #10 (due Fri, 30 Jan)
3.3 # 11-31 odd - Increasing, Decreasing, and Relative Extrema
3.4 #11-25 odd - Inflection Points and Concavity
3.5 #15-31 odd - Limits at Infinity
Quiz #7 - Fri, 30 Jan

HW #11 (due Wed, 4 Feb)
7.7 #11-35 odd - l'Hopital's Rule
7.7 #37-53 odd - l'Hopital's Rule with logs
3.7 #3,5,17,23,29 - Optimization
Test 4 - Wed, 4 Feb

Final Exam - Thurs, 12 Feb 1:00-3:00pm

3.5 Limits at Infinity

$\lim_{x \to \infty} f(x)$ (end behavior)

correspond exactly with horizontal & oblique asymptotes

$$f(x)=\frac{5x^2-3x+4}{2x^2+5x}\approx\frac{5x^2}{2x^2}=\frac{5}{2}$$

Horizontal asymptote $y=5/2$

$$\lim_{x\to\pm\infty} f(x)=\frac{5}{2}$$

$$f(x)=\frac{2x-4}{3x^4}\approx\frac{2x}{3x^4}=\frac{2}{3x^3}\longrightarrow 0$$

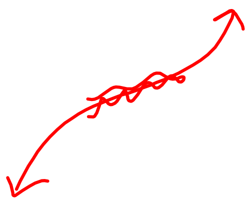
as $x\to\pm\infty$

$$\lim_{x\to\pm\infty} f(x)=0$$

Horizontal asymptote

$y=0$

$$f(x)=\frac{2x^7-4x^3-2}{5x^4+1}\approx\frac{2x^7}{5x^4}=\frac{2}{5}x^3$$

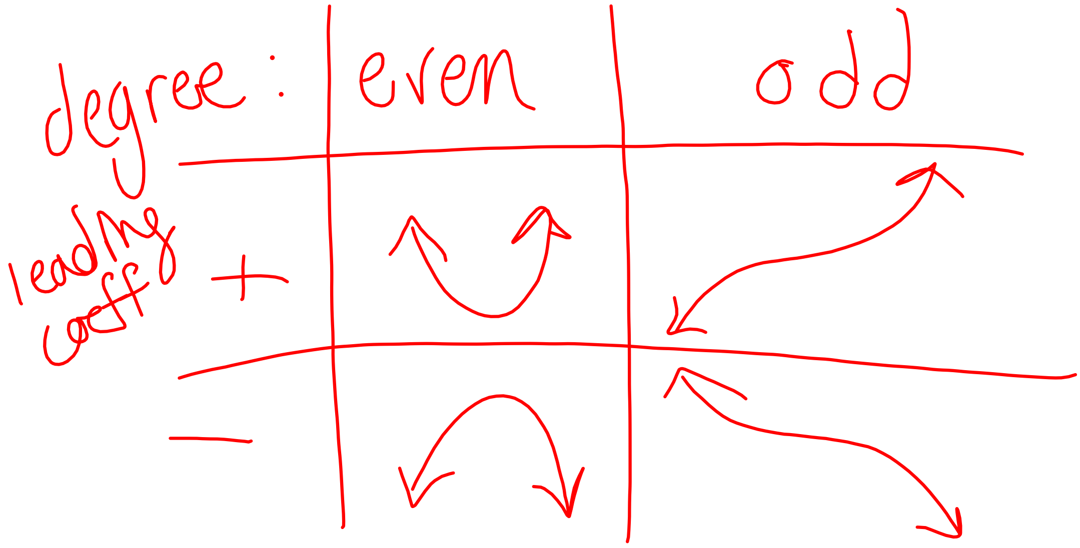


$$\lim_{x\to\infty} f(x)=\infty$$

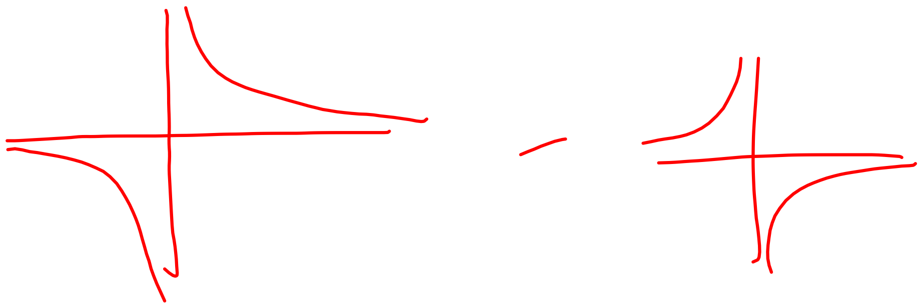
$$\lim_{x\to-\infty} f(x)=-\infty$$

$$f(x)=\frac{2-7x^3+2x}{1+x}\approx\frac{-7x^3}{x}=-7x^2$$

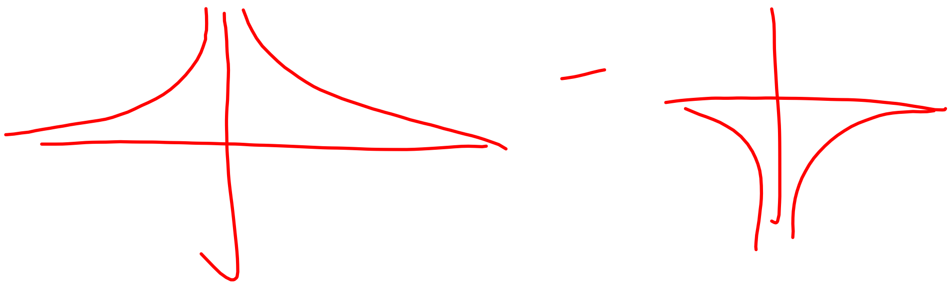
$$\lim_{x\to\pm\infty} f(x)=-\infty$$



$\frac{1}{x^{odd}}$



$\frac{1}{x^{even}}$



25. $\lim\limits_{x\to-\infty}\left(\frac{1}{2}x - \frac{4}{x^2}\right)$

$= \lim\limits_{x\to-\infty}\frac{1}{2}x - \lim\limits_{x\to-\infty}\frac{4}{x^2}$

$= -\infty - 0$

$= \boxed{-\infty}$

26. $\lim\limits_{x\to-\infty}\frac{x}{\sqrt{x^2+1}}$

$= \lim\limits_{x\to-\infty}\frac{x}{\sqrt{x^2}}$

$= \lim\limits_{x\to-\infty}\frac{x}{|x|}$

$= \lim\limits_{x\to-\infty}\frac{x}{-x} = \boxed{-1}$

$\sqrt[n]{x^n} = \begin{cases} x, & n \text{ odd} \\ |x|, & n \text{ even} \end{cases}$

$\sqrt{(-3)^2} = \sqrt{9} = 3 = |-3|$

$\sqrt[3]{(-2)^3} = \sqrt[3]{-8} = -2$

$|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$

$$\lim_{x\to\infty} \frac{5x-2}{\sqrt{9x^2+3}} = \lim_{x\to\infty} \frac{5x}{\sqrt{9x^2}}$$

$$= \lim_{x\to\infty} \frac{5x}{|3x|} = \lim_{x\to\infty} \frac{5x}{3x} = \boxed{\frac{5}{3}}$$

30. $$\lim_{x\to\infty} \frac{x-\cos x}{x}$$

$$\frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}$$

$$= \lim_{x\to\infty} \frac{x}{x} - \lim_{x\to\infty} \frac{\cos x}{x}$$

$$= 1 - \lim_{x\to\infty} \frac{\cos x}{x}$$ ← bounded by [-1, 1]

$x \to \infty$

$$= 1 - 0 = \boxed{1}$$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x\to 0} \frac{1-\cos x}{x} = 0$$

32. $\lim\limits_{x\to\infty} \cos\frac{1}{x} = \cos\left[\lim\limits_{x\to\infty} \frac{1}{x}\right]$

$= \cos(0) = \boxed{1}$

18. c.

$\lim\limits_{x\to\infty} \frac{5x^{3/2}}{4\sqrt{x}+1} = \lim\limits_{x\to\infty} \frac{5x^{3/2}}{4x^{1/2}+1}$

$= \lim\limits_{x\to\infty} \frac{5x^{3/2}}{4x^{1/2}} = \lim\limits_{x\to\infty} \frac{5x}{4} = \boxed{\infty}$

3.7 Optimization Problems

4. Find two positive numbers whose product is 192 and the sum of the first plus three times the second is a minimum.

x & y

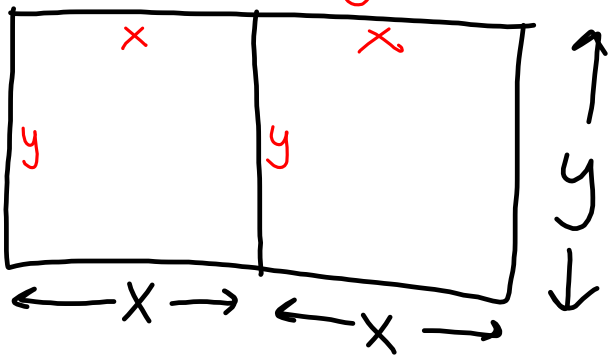
$xy = 192 \Rightarrow y = \frac{192}{x}$

$$S(x) = x + 3\left(\frac{192}{x}\right)$$
$$= x + 3(192)x^{-1}$$
$$S'(x) = 1 - 3(192)x^{-2}$$
$$1 - \frac{3(192)}{x^2} = 0$$
$$1 = \frac{3(192)}{x^2} \Rightarrow x^2 = 3(192)$$
$$x = \sqrt{3(192)} = 24$$

$$\boxed{\begin{array}{l} x = 24 \\ y = 8 \end{array}}$$

18. A rancher has 200 feet of fencing with which to enclose two adjacent corrals, arranged according to the figure. What dimensions should be used so that the enclosed area will be a maximum?

$A = 2xy$ $\quad 4x + 3y = 200$



$$3y = 200 - 4x$$
$$y = \frac{200}{3} - \frac{4}{3}x$$
$$A(x) = 2x\left(\frac{200}{3} - \frac{4}{3}x\right)$$
$$A(x) = \frac{400}{3}x - \frac{8}{3}x^2$$
$$A'(x) = \frac{400}{3} - \frac{16}{3}x$$

$$\frac{400}{3} - \frac{16}{3}x = 0$$
$$\frac{3}{16} \cdot \frac{400}{3} = \frac{16}{3}x \cdot \frac{3}{16}$$
$$\boxed{25 = x}$$

$$\boxed{y = \frac{100}{3}}$$