HW #9 (due Fri, 23 Jan)
3.1 # 17-31 odd - Absolute Extrema on an Interval
3.2 # 7-19 odd - Rolle's Theorem
3.2 # 31-37 odd - Mean Value Theorem
Quiz #6 - Mon, 26 Jan

HW #10 (due Fri, 30 Jan)
3.3 # 11-31 odd - Increasing, Decreasing, and Relative Extrema
3.4 #11-25 odd - Inflection Points and Concavity
3.5 #15-31 odd - Limits at Infinity
Quiz #7 - Fri, 30 Jan

HW #11 (due Wed, 4 Feb)
3.7 #3,5,17,23,29 - Optimization
7.7 #11-35 odd - l'Hopital's Rule
7.7 #37-53 odd - l'Hopital's Rule with logs
Test 4 - Wed, 4 Feb

Final Exam - Thurs, 12 Feb 1:00-3:00pm

7.7 Indeterminate Forms & L'Hôpital's Rule

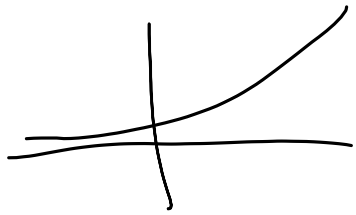
$\boxed{\frac{0}{0}, \quad \frac{\infty}{\infty},}$ $0 \cdot \infty, \quad 1^{\infty}, \quad 0^{0}, \quad and \quad \infty - \infty$ are called indeterminate forms.

L'Hôpital's Rule:

Let f and g be functions that are differentiable on an open interval (a, b) containing c, except possibly at c itself. Assume that $g'(x) \neq 0$ for all x in (a, b), except possibly at c itself. If the limit of f(x)/g(x) as x approaches c produces an indeterminate form $0/0$, ∞/∞, $(-\infty)/\infty$, or $\infty/(-\infty)$, then

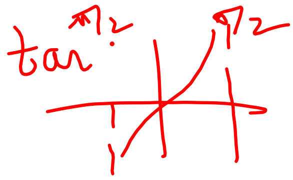
$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

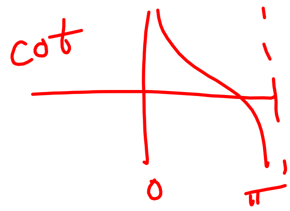
36. $\lim\limits_{x\to\infty} \dfrac{e^{x/2}}{x} = \dfrac{\infty}{\infty}$



$= \lim\limits_{x\to\infty} \dfrac{e^{x/2}\cdot\frac{1}{2}}{1} = \boxed{\infty}$

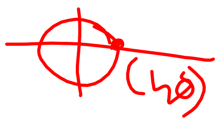
38. $\lim\limits_{x\to 0^+} x^3 \cot x = 0\cdot\infty$





$= \lim\limits_{x\to 0^+} \dfrac{x^3}{\tan x} = \dfrac{0}{0}$

$= \lim\limits_{x\to 0^+} \dfrac{3x^2}{\sec^2 x} = \dfrac{0}{1} = \boxed{0}$



40. $\lim\limits_{x\to\infty} x\tan\frac{1}{x} = \infty\cdot 0$

$= \lim\limits_{x\to\infty} \dfrac{\tan\frac{1}{x}}{\frac{1}{x}} = \frac{0}{0}$

$= \lim\limits_{x\to\infty} \dfrac{\sec^2\left(\frac{1}{x}\right)\cdot \cancel{\frac{-1}{x^2}}}{\cancel{\frac{-1}{x^2}}} = \boxed{1}$

42. $\lim\limits_{x\to 0^+} \left(e^x + x\right)^{2/x} = 1^\infty$

$y = \lim\limits_{x\to 0^+} \left(e^x + x\right)^{2/x}$

$\ln y = \ln\left[\lim\limits_{x\to 0^+} \left(e^x + x\right)^{2/x}\right]$

$\ln y = \lim\limits_{x\to 0^+} \left[\ln\left(e^x + x\right)^{2/x}\right]$

$\ln y = \lim\limits_{x\to 0^+} \left[\frac{2}{x}\ln\left(e^x + x\right)\right]$

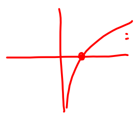
$\ln y = \lim\limits_{x\to 0^+} \dfrac{\ln\left(e^x + x\right)}{x/2}$

$\ln y = \lim\limits_{x\to 0^+} \dfrac{\frac{1}{e^x+x}\cdot\left(e^x+1\right)}{1/2}$

$\ln y = 4$

$e^{\ln y} = e^4$

$\boxed{y = e^4}$



$\boxed{\log_a b^p = p\cdot\log_a b}$

$\dfrac{\ln(e^x+x)}{\frac{x}{2}}$ $\dfrac{\frac{2}{x}}{\frac{1}{\ln(e^x+x)}}$

$$\lim_{x\to\infty} \sin\frac{1}{x}$$

$$= \sin\left[\lim_{x\to\infty} \frac{1}{x}\right]$$

$$= \sin 0$$

$$= 0$$

44. $\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x \quad = 1^\infty$

$$y = \lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x$$

$$\ln y = \ln\left[\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x\right]$$

$$\ln y = \lim_{x\to\infty} \left[\ln\left(1+\frac{1}{x}\right)^x\right]$$

$$\ln y = \lim_{x\to\infty} \left[x \ln\left(1+\frac{1}{x}\right)\right]$$

$$\ln y = \lim_{x\to\infty} \frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}}$$

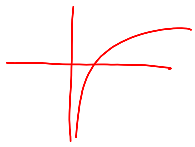
$$\ln y = \lim_{x\to\infty} \frac{\frac{1}{1+\frac{1}{x}}\cdot\cancel{\left(\frac{-1}{x^2}\right)}}{\cancel{\frac{-1}{x^2}}}$$

$$\ln y = 1$$

$$e^{\ln y} = e^1$$

$$\boxed{y = e}$$

$$\lim_{x\to0^+}(\sin x)^x$$



$$y = \lim_{x\to0^+}(\sin x)^x$$

$$\ln y = \lim_{x\to0^+}\left[x\,\underbrace{\ln(\sin x)}_{-\infty}\right]$$

$$\ln y = \lim_{x\to0^+}\frac{\ln(\sin x)}{\frac{1}{x}}$$

$$\ln y = \lim_{x\to0^+}\frac{\frac{1}{\sin x}\cdot\cos x}{\frac{-1}{x^2}}$$

$$\ln y = \lim_{x\to0^+}\frac{-x^2}{\tan x}$$

$$\ln y = \lim_{x\to0^+}\frac{-2x}{\sec^2 x}$$

$$\ln y = 0$$

$$y = e^0$$

$$\boxed{y = 1}$$