Informal Description of the Limit

If *f(x)* becomes arbitrarily close to a single number *L* as *x* approaches *c* from either side, the limit of *f(x)*, as *x* approaches *c*, is *L*.

$$\lim_{x\to c} f(x) = L$$

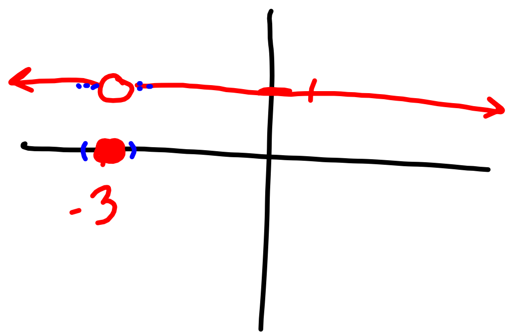
Note: the existence or nonexistence of *f(x)* at *x=c* has no bearing on the existence of the limit as *x* approaches *c*.

$$f(x) = \begin{cases} 1, & x \neq -3 \\ 0, & x = -3 \end{cases}$$

$y=1$

$f(-3)=0$

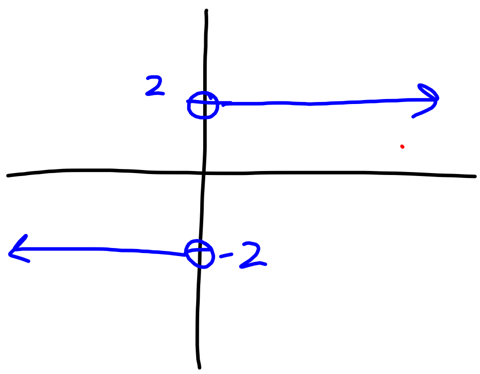
$$\lim_{x\to -3} f(x) = \boxed{1}$$



$$\lim_{x \to 0} \frac{|2x|}{x}$$

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$\frac{|2x|}{x} = \begin{cases} \frac{2x}{x} = 2, & x > 0 \\ \frac{-(2x)}{x} = -2, & x < 0 \end{cases}$$



$\lim_{x \to 0} f(x) = ?$

Does not exist

$\lim_{x \to 0^+} f(x) = 2$

$\lim_{x \to 0^-} f(x) = -2$

$$\lim_{x \to 3} \frac{|x-3|}{x-3}$$ Does not exist

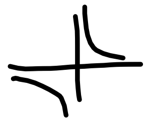
$$\frac{|x-3|}{x-3} = \begin{cases} \frac{x-3}{x-3} = 1, & x-3 > 0 \\ & x > 3 \\ \frac{-(x-3)}{x-3} = -1, & x-3 < 0 \\ & x < 3 \end{cases}$$

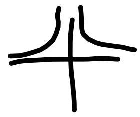
$\lim_{x \to 3^-} f(x) = \boxed{-1}$

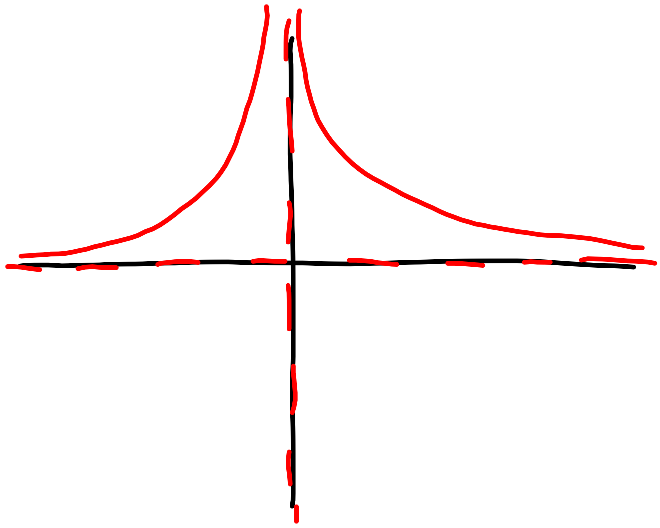
$\lim_{x \to 3^+} f(x) = \boxed{1}$

$\lim_{x \to 7} \frac{|x-3|}{x-3} = \boxed{1}$

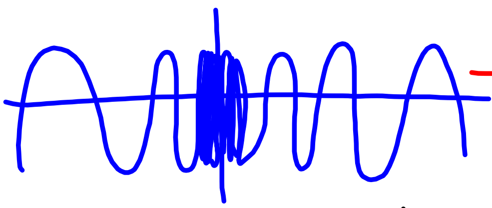
$$\lim_{x \to 0} \frac{1}{x^4} = \boxed{\infty}$$

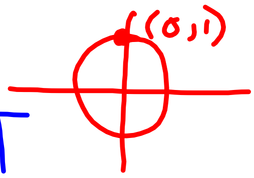
$\frac{1}{x^{odd}}$ 

$\frac{1}{x^{even}}$ 



$$\lim_{x \to 0} \sin \frac{1}{x}$$





x	$\frac{2}{\pi}$	$\frac{2}{3\pi}$	$\frac{2}{5\pi}$	$\frac{2}{7\pi}$	$\frac{2}{9\pi}$	$\frac{2}{11\pi}$
$\sin\frac{1}{x}$	1	-1	1	-1	1	-1

The Limit Does not exist

"Dirichlet Function"

$$f(x) = \begin{cases} 0 , & \text{if } x \text{ is rational} \\ 1 , & \text{if } x \text{ is irrational} \end{cases}$$

$\lim_{x \to c} f(x)$ does not exist for any value of c.

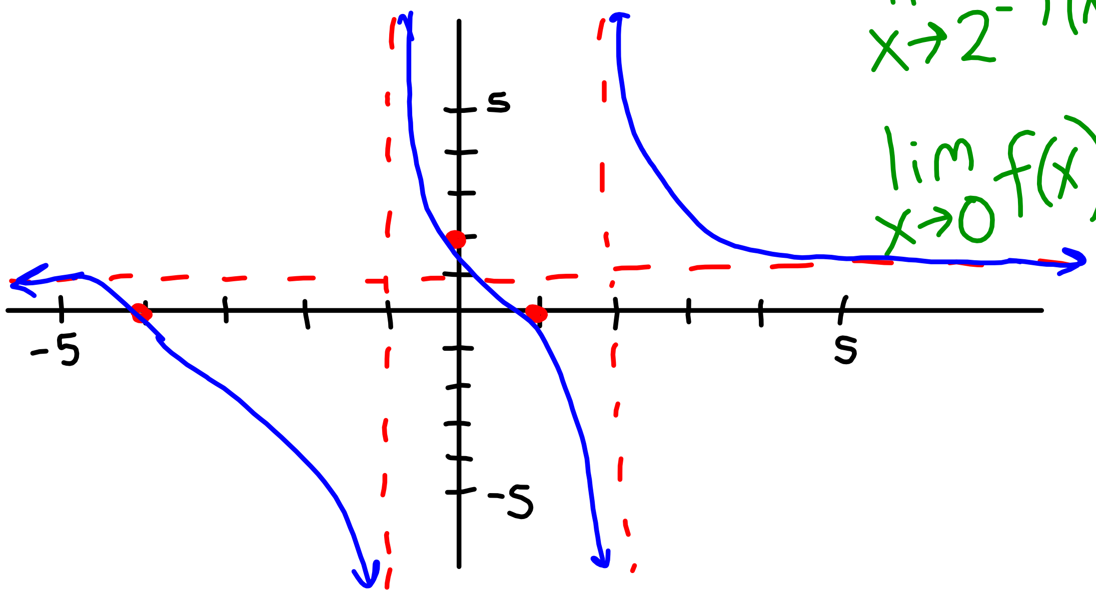
Graph the rational function.

$$f(x) = \frac{(x+4)(x-1)}{(x-2)(x+1)}$$

$\lim_{x \to \infty} f(x) = 1$

$\lim_{x \to 2^-} f(x) = -\infty$

$\lim_{x \to 0} f(x) = 2$

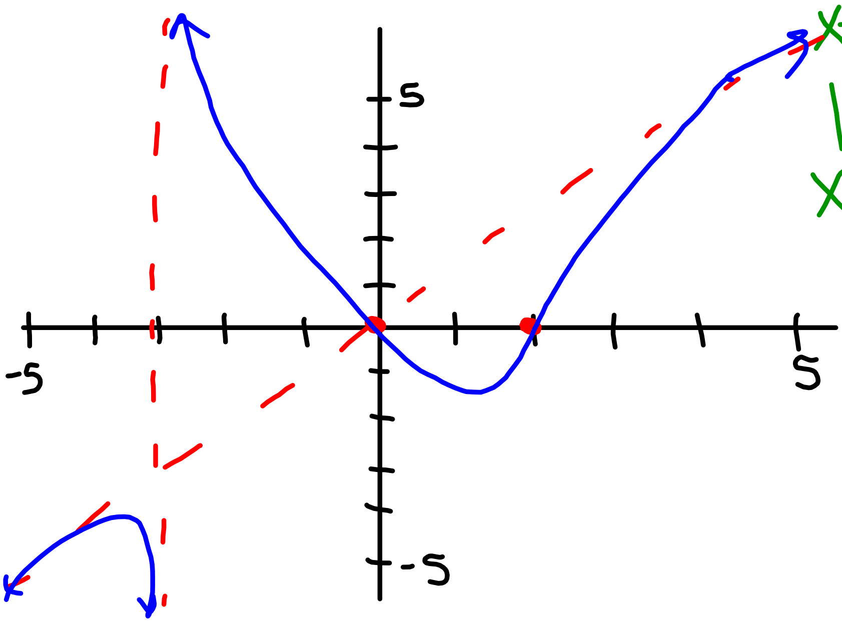


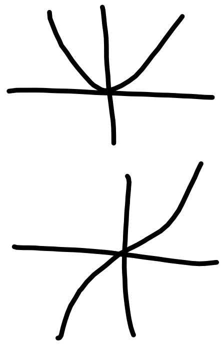
$$f(x) = \frac{x(x-2)}{x+3}$$

$$\lim_{x \to -\infty} f(x) = \boxed{-\infty}$$

$$\lim_{x \to -3^+} f(x) = \boxed{\infty}$$

$$\lim_{x \to 2} f(x) = \boxed{0}$$





<u>Informal Description of the Limit</u>

If *f(x)* becomes arbitrarily close to a single number *L* as *x* approaches *c* from either side, the limit of *f(x)*, as *x* approaches *c*, is *L*.

$$\lim_{x \to c} f(x) = L$$

<u>Note</u>: the existence or nonexistence of *f(x)* at *x=c* has no bearing on the existence of the limit as *x* approaches *c*.

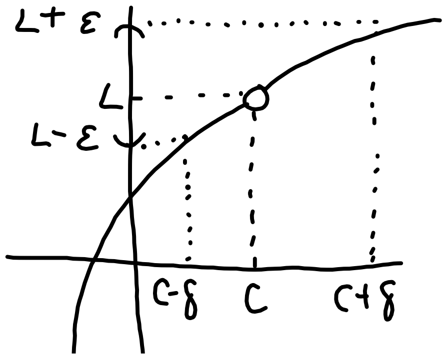
Building up to the $\epsilon - \delta$ Definition of the Limit

ε = epsilon

δ = delta

Translating the "informal description": $\lim_{x\to c} f(x) = L$

If $f(x)$ becomes arbitrarily close to a single number L as x approaches c from either side, the limit of $f(x)$, as x approaches c, is L.



"$f(x)$ becomes arbitrarily close to L"

$f(x)$ lies in the interval $(L-\varepsilon, L+\varepsilon)$ for some (really small) $\varepsilon > 0$.

$|f(x) - L| < \varepsilon$

"the distance between $f(x)$ and L is less than ε"

"x approaches c"

There exists a (very small) positive number δ such that x is either in the interval $(c-\delta, c)$ or $(c, c+\delta)$.

$0 < |x - c| < \delta$

The first inequality guarantees that $x \neq c$.

$\varepsilon - \delta$ Definition of the Limit:

Let f be a function defined on an open interval containing c (except possibly at c) and let L be a real number. The statement

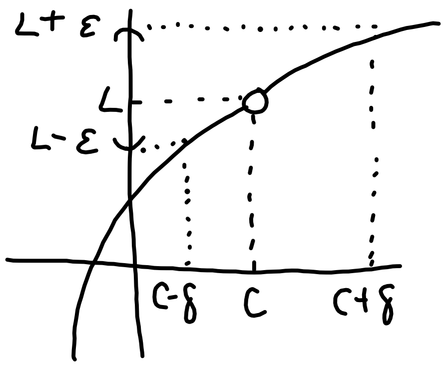
$$\lim_{x\to c} f(x) = L$$

The limit as x approaches c of f(x) is L, if

Given a small # ε, we can find another small # δ such that

means that for each $\varepsilon > 0$, there exists a $\delta > 0$ such that if $0 < |x - c| < \delta$, then $|f(x) - L| < \varepsilon$.

If x is δ-close to c, then f(x) is ε-close to L.



Homework:

Already assigned:
1.2 #1-7odd,9-18all

New:
~~1.2 #23, 25, 27, 29, 30, 31~~
and watch all of the Khan Academy epsilon-delta videos!