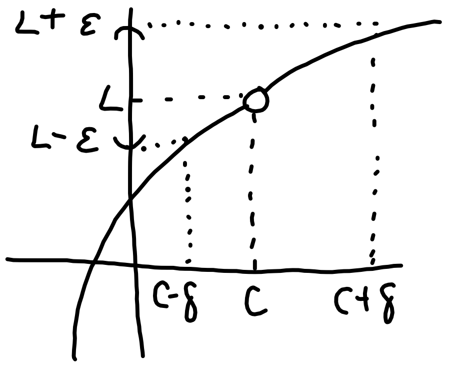
## $\varepsilon - \delta$ Definition of the Limit:

Let $f$ be a function defined on an open interval containing $c$ (except possibly at $c$) and let $L$ be a real number. The statement

$$\lim_{x \to c} f(x) = L$$

means that for each $\varepsilon > 0$, there exists a $\delta > 0$ such that if $0 < |x - c| < \delta$, then $|f(x) - L| < \varepsilon$.

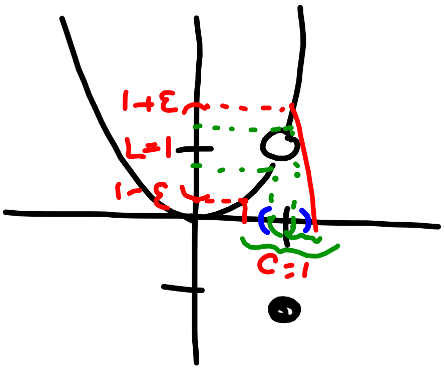



## $\varepsilon - \delta$ Definition of the Limit:

$\lim_{x \to c} f(x) = L$ if given $\varepsilon > 0$, there exists a $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.

$$f(x) = \begin{cases} x^2, & x \neq 1 \\ -1, & x = 1 \end{cases}$$

Find $\lim_{x \to 1} f(x) = 1$



**$\varepsilon - \delta$ Definition of the Limit:**

$\lim_{x\to c} f(x) = L$ if given $\varepsilon > 0$, there exists a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.

$f(x) = 2x - 1$ $\quad L = 7$ ; $c = 4$

Find $\lim_{x\to 4} f(x)$ and prove that is the limit using the $\varepsilon$-$\delta$ definition.

Let $\varepsilon > 0$ be given.

We want to find a $\delta > 0$ such that whenever $|x-4| < \delta$, we get $|(2x-1)-7| < \varepsilon$.

Start w/ $|f(x)-L| < \varepsilon$

$|2x-8| < \varepsilon \Rightarrow |2(x-4)| < \varepsilon \Rightarrow 2|x-4| < \varepsilon$

$\Rightarrow |x-4| < \varepsilon/2$. Take $\delta = \varepsilon/2$.

Then whenever $|x-c| < \delta$ $\quad |x-4| < \varepsilon/2$,

We have that $|f(x)-L| = |2x-1-7| = |2x-8| =$

$= 2|x-4| < 2 \cdot \varepsilon/2 = \varepsilon$.

Therefore $\lim_{x\to 4}(2x-1)$ is indeed 7.

**$\varepsilon - \delta$ Definition of the Limit:**

$\lim_{x\to c} f(x) = L$ if given $\varepsilon > 0$, there exists a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.

$f(x) = -5x + 3$ ; find $\lim_{x\to 1} f(x)$ & find a $\delta$.

$L = -5(1) + 3 = -2$ $\quad$ ; $c = 1$

Let $\varepsilon > 0$ be given.

$\boxed{|f(x) - L| < \varepsilon}$

$|-5x + 3 - (-2)| < \varepsilon$

$|-5x + 5| < \varepsilon$

$|-5(x-1)| < \varepsilon$

$5|x-1| < \varepsilon$

$|x-1| < \varepsilon/5$

$\boxed{|x-c| < \{\ \}}$

Take $\boxed{\delta = \varepsilon/5}$

Prove that the limit is $L$ using the $\varepsilon - \delta$ definition of the limit.

28. $\lim_{x \to -3} (2x + 5) = 2(-3) + 5 = -1$

$|2x+5-(-1)| < \varepsilon$

$|2x+6| < \varepsilon$

$2|x+3| < \varepsilon$

$|x+3| < \varepsilon/2$

$\boxed{\text{Take } \delta = \varepsilon/2}$

$|x-(-3)|$

Proof: Let $\varepsilon > 0$ be given.
We want to find a $\delta > 0$ such that whenever $|x-c| < \delta$, we get $|f(x) - L| < \varepsilon$.
Take $\delta = \varepsilon/2$.
Then whenever $|x-(-3)| < \varepsilon/2$, we have that $|2x+5-(-1)| = |2x+6| = 2|x+3| < 2 \cdot \varepsilon/2 = \varepsilon$.
Hence $\lim_{x \to -3} (2x+5)$ is indeed $-1$. $\square$

Find $\delta$ for $\boxed{\varepsilon = 0.01}$

24. $\lim_{x \to 4} \left(4 - \frac{x}{2}\right) = 4 - \frac{4}{2} = 4 - 2 = \boxed{2 = L}$

$\boxed{c = 4}$

$|f(x) - L| < \varepsilon$

$\left|4 - \frac{x}{2} - 2\right| < 0.01$

$\left|2 - \frac{x}{2}\right| < 0.01$

$\left|-\frac{1}{2}(x-4)\right| < 0.01$

$\frac{1}{2}|x-4| < 0.01$

$|x-4| < 0.02$

$\boxed{\delta = 0.02}$

Find $\delta$ for $\boxed{\varepsilon = 0.01}$

26. $\lim\limits_{x\to 5}(x^2+4) = 5^2+4 = \boxed{29 = L}$

$\boxed{c=5}$

$|f(x)-L| < \varepsilon \rightsquigarrow |x-c| < \delta$

$|x^2+4-29| < \varepsilon$

$|x^2-25| < \varepsilon$

4 5 6

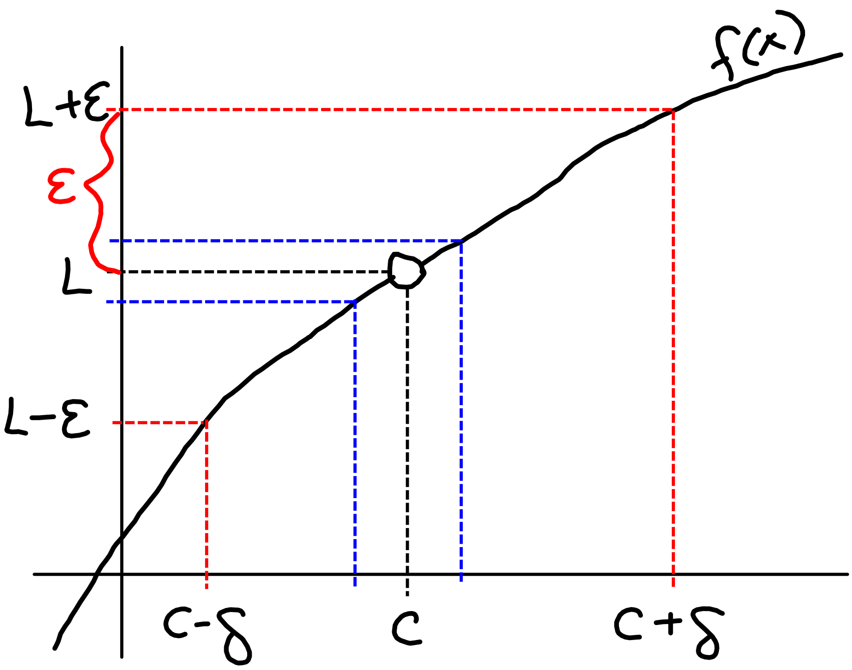
$|(x+5)(x-5)| < 11|x-5| < \varepsilon$

really close to 5, $x<6$

so $x+5<11$

$|x-5| < \varepsilon/11$

$\boxed{\delta = \varepsilon/11} = \boxed{\frac{0.01}{11}}$



## Homework:

- 1.2 #1-7odd,9-18all
- **1.2 #23, 25, 27, 29, 30, 31**