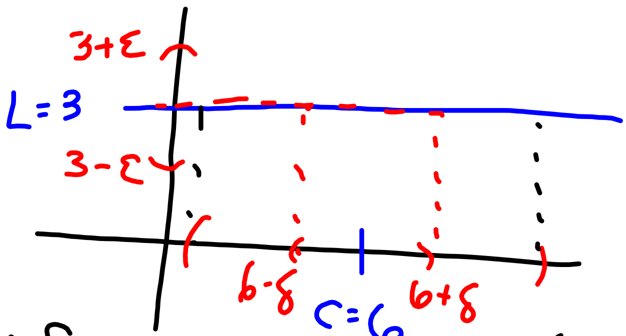
Turn in HW #1:

1.2 #1-7odd,9-18all
1.2 #23, 25, 27, 29, 30, 31

$$\lim_{x\to 6}(3) = 3$$

When to have 1st quiz?

Wed. 8/20



Let $\varepsilon > 0$ be given. Take $\delta = \varepsilon$.

Take $\delta = \varepsilon$

Then whenever $|x-6| < \delta$,

we have $|f(x) - L| = |3-3| = 0 < \varepsilon$.

Hence $\lim_{x\to 6} 3 = 3$. Yay!

1.3 Evaluating Limits Analytically

If $\lim_{x\to c} f(x) = f(c)$,

we say that $f(x)$ is

continuous at c.

Basic Limits

$a, c \in \mathbb{R}$

$n \in \mathbb{N}$

$$\lim_{x \to c} a = a$$

$$\lim_{x \to 5} (-3) = -3$$

$$\lim_{x \to c} x = c$$

$$\lim_{x \to -\pi} x = -\pi$$

$$\lim_{x \to c} x^n = c^n$$

$$\lim_{x \to -1} x^5 = -1$$

Theorem 1.2 more properties of Limits

$b, c \in \mathbb{R}$, $n > 0$ an integer, f & g - functions

$\lim_{x \to c} f(x) = L$; $\lim_{x \to c} g(x) = K$

1. scalar multiple
$$\lim_{x \to c} [b f(x)] = bL$$

2. sum or difference
$$\lim_{x \to c} [f(x) \pm g(x)] = L \pm K$$

3. product
$$\lim_{x \to c} [f(x) g(x)] = LK$$

4. quotient
$$\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{K}, \quad K \neq 0$$

5. power
$$\lim_{x \to c} [f(x)]^n = L^n$$
(follows from #3)

polynomials, rational functions, $\sqrt[n]{x}$, $f(g(x))$, sin, cos, etc.

$$\lim_{x\to c}\left[f(x) \pm g(x)\right] = \lim_{x\to c} f(x) \pm \lim_{x\to c} g(x)$$

$$\lim_{x\to c}\left[f(x)g(x)\right] = \left[\lim_{x\to c} f(x)\right]\cdot\left[\lim_{x\to c} g(x)\right]$$

$$\lim_{x\to c}\frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}, \quad \lim_{x\to c} g(x) \neq 0$$

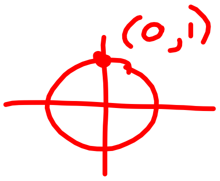
$$\lim_{x\to c}\left[f(x)\right]^n = \left[\lim_{x\to c} f(x)\right]^n$$

1.3

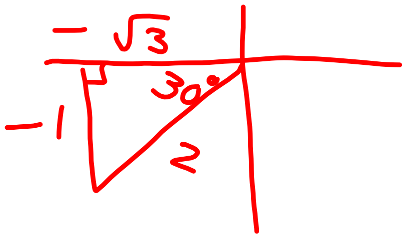
12. $\lim_{x\to 1}\left(3x^3 - 2x^2 + 4\right) = \boxed{5}$

18. $\lim_{x\to 3}\frac{\sqrt{x+1}}{x-4} = \frac{\sqrt{3+1}}{3-4} = \frac{\sqrt{4}}{-1} = \boxed{-2}$

30. $\lim\limits_{x \to 1} \sin \dfrac{\pi x}{2} = \sin \dfrac{\pi}{2} = \boxed{1}$



36. $\lim\limits_{x \to 7} \sec\left(\dfrac{\pi x}{6}\right) = \sec \dfrac{7\pi}{6} = \boxed{-\dfrac{2}{\sqrt{3}}}$



38. $\lim\limits_{x \to c} f(x) = \dfrac{3}{2}$; $\lim\limits_{x \to c} g(x) = \dfrac{1}{2}$

(a) $\lim\limits_{x \to c} \left[4 f(x)\right] = 4 \cdot \lim\limits_{x \to c} f(x) = 4 \cdot \dfrac{3}{2} = \boxed{6}$

(b) $\lim\limits_{x \to c} \left[f(x) + g(x)\right] = \dfrac{3}{2} + \dfrac{1}{2} = \boxed{2}$

(c) $\lim\limits_{x \to c} \left[f(x) g(x)\right] = \dfrac{3}{2} \cdot \dfrac{1}{2} = \boxed{\dfrac{3}{4}}$

(d) $\lim\limits_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{3/2}{1/2} = \dfrac{3}{2} \cdot \dfrac{2}{1} = \boxed{3}$

$$\lim_{x \to 3} \frac{x^2-2x-3}{x-3} = \lim_{x \to 3} \frac{\cancel{(x-3)}(x+1)}{\cancel{x-3}}$$

$$= \lim_{x \to 3} (x+1) = \boxed{4}$$

$$\lim_{x \to 4} \frac{x-4}{\sqrt{x}-2} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2}$$

$$= \lim_{x \to 4} \frac{\cancel{(x-4)}(\sqrt{x}+2)}{\cancel{x-4}}$$

$$= \lim_{x \to 4} (\sqrt{x}+2) = \boxed{4}$$

Given $f(x) = 2x^2 + 3x + 1$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{2(x+h)^2 + 3(x+h) + 1 - (2x^2 + 3x + 1)}{h}$$

$$= \lim_{h\to 0} \frac{2(x^2 + 2xh + h^2) + 3x + 3h + 1 - 2x^2 - 3x - 1}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 + \cancel{3x} + 3h \cancel{+1} \cancel{-2x^2} \cancel{-3x} \cancel{-1}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(4x + 2h + 3)}{\cancel{h}} = \lim_{h\to 0} (4x + 2h + 3) = \boxed{4x + 3}$$

$f(x) = x^3$

$(a+b)^3$
$a^3 + 3a^2b + 3ab^2 + b^3$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h\to 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h\to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 \cancel{-x^3}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}} = \lim_{h\to 0} (3x^2 + 3xh + h^2)$$

$$= \boxed{3x^2}$$

1.3 Evaluating Limits Analytically

42. $h(x) = \dfrac{x^2 - 3x}{x}$

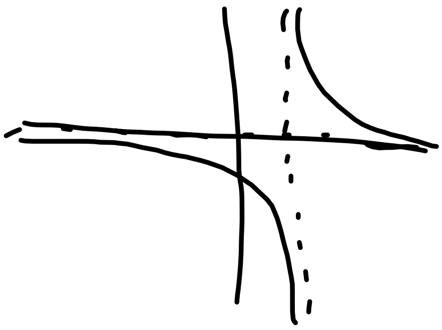
(a) $\lim\limits_{x \to -2} h(x) = \dfrac{(-2)^2 - 3(-2)}{-2} = \dfrac{4+6}{-2} = \dfrac{10}{-2} = \boxed{-5}$

(b) $\lim\limits_{x \to 0} h(x) = \lim\limits_{x \to 0} \dfrac{\cancel{x}(x-3)}{\cancel{x}} = 0-3 = \boxed{-3}$

44. $\lim\limits_{x \to 1} \dfrac{x}{x^2 - x} = \lim\limits_{x \to 1} \dfrac{\cancel{x}}{\cancel{x}(x-1)}$

$= \lim\limits_{x \to 1} \dfrac{1}{x-1}$

$\boxed{\text{Does not exist}}$



48. $\lim_{x \to -1} \frac{x^3+1}{x+1}$

$a^3+b^3 = (a+b)(a^2-ab+b^2)$

$= \lim_{x \to -1} \frac{\cancel{(x+1)}(x^2-x+1)}{\cancel{x+1}}$

$= \lim_{x \to -1} (x^2-x+1) = (-1)^2-(-1)+1 = \boxed{3}$

Homework #2 - due next Fri, 8/22:

1.3 #11,17,27-35odd, 39-61odd

Quiz #1
Wed 8/20