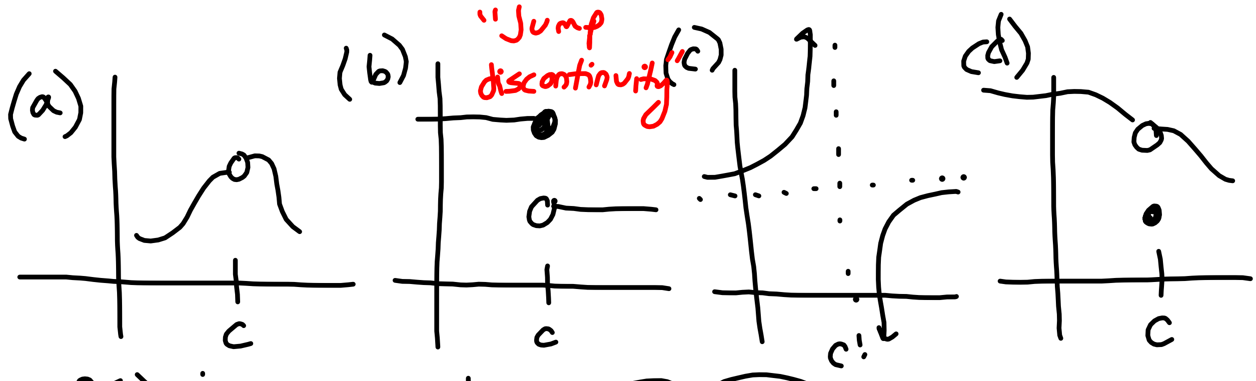
1.4 Continuity and One-Sided Limits

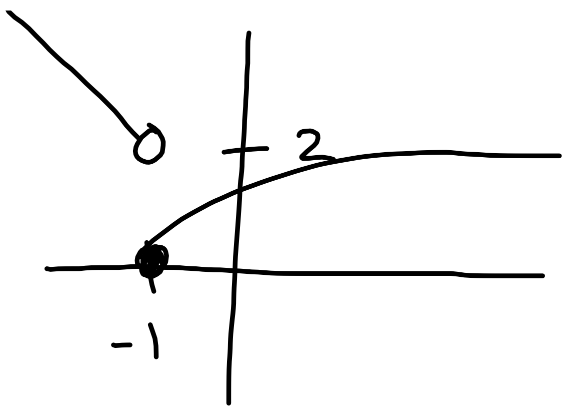


(a) $f(c)$ is undefined

(b), (c) $\lim_{x \to c} f(x)$ does not exist

(d) $\lim_{x \to c} f(x) \neq f(c)$

These are all discontinuities
(a) and (d) are removable
(b) and (c) are nonremovable



$$\lim_{x \to -1^-} f(x) = 2$$

$$\lim_{x \to -1^+} f(x) = 0$$

$$\lim_{x \to -1} f(x) = \text{does not exist}$$

One-Sided Limits

$\lim_{x \to c^+} f(x) = L$ *limit from the right*

$\lim_{x \to c^-} f(x) = L$ *limit from the left*

$$\lim_{x \to c} f(x) = L \textit{ if and only if}$$

$$\lim_{x \to c^-} f(x) = L = \lim_{x \to c^+} f(x)$$

Continuity at a point

A function f is continuous at c if the following 3 conditions are met:

1. $f(c)$ is defined
2. Limit of $f(x)$ exists when x approaches c
3. Limit of $f(x)$ when x approaches c is equal to $f(c)$

f(x) is continuous at c if

$$\lim_{x \to c} f(x) = f(c)$$

Continuity on an open interval

A function is continuous on an open interval if it is continuous at each point in the interval. A function that is continuous on the entire real line $(-\infty, \infty)$ is everywhere continuous.

Continuity on a closed interval

A function f is continuous on the closed interval $[a, b]$ if it is continuous on the open interval I (a, b) and $\lim_{x \to a^+} f(x) = f(a)$ and $\lim_{x \to b^-} f(x) = f(b)$.

$[\leftarrow \quad \rightarrow]$

10. $$\lim_{x \to 4^-} \frac{\sqrt{x} - 2}{x - 4} \cdot \frac{\sqrt{x} + 2}{\sqrt{x} + 2}$$

$$= \lim_{x \to 4^-} \frac{\cancel{x-4}^{\,1}}{\cancel{(x-4)}(\sqrt{x}+2)} =$$

$$= \lim_{x \to 4^-} \frac{1}{\sqrt{x}+2} = \frac{1}{\sqrt{4}+2} = \boxed{\frac{1}{4}}$$

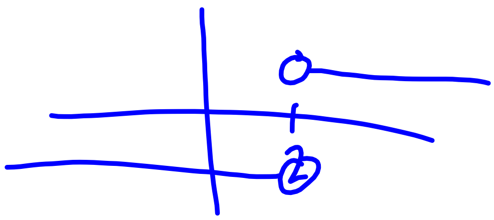
12. $\lim\limits_{x\to 2^+} \dfrac{|x-2|}{x-2} = \boxed{1}$

$|x| = \begin{cases} x, & x \ge 0 \\ -x, & x < 0 \end{cases}$

$$\frac{|x-2|}{x-2} = \begin{cases} \dfrac{x-2}{x-2} = 1, & x-2>0,\ x>2 \\ \dfrac{-(x-2)}{x-2} = -1, & x-2<0,\ x<2 \end{cases}$$

$|x-2| = \begin{cases} (x-2), & x-2 \ge 0 \\ -(x-2), & x-2 < 0 \end{cases}$

$$= \begin{cases} 1, & x>2 \\ -1, & x<2 \end{cases}$$



1.4

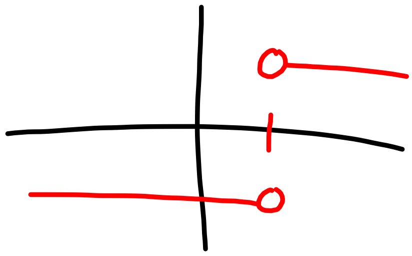
Discuss the [dis]continuity of the function.

$$f(x) = \frac{(x+4)(x-2)}{(x-2)(x+1)}$$

V.A.: $x=-1$
hole: $x=2$

removable discontinuity @ 2
non-removable discontinuity @ -1
f is continuous on: $(-\infty,-1)\cup(-1,2)\cup(2,\infty)$

$$f(x) = \frac{|x-2|}{x-2}$$



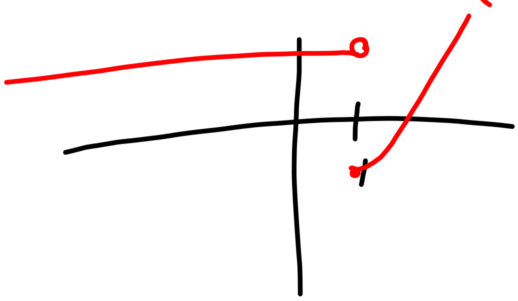
non-removable discontinuity @ 2

continuous on $(-\infty, 2) \cup (2, \infty)$

$$f(x) = \begin{cases} x^2 - 2, & x \geq 1 \\ 5, & x < 1 \end{cases}$$

non removable discontinuity @ 1

continuous on $(-\infty, 1) \cup (1, \infty)$



$$f(x)=\begin{cases} x+6, & x\le -2 \\ x^2, & -2<x\le 3 \\ 8 & x>3 \end{cases}$$

$-2+6=4$

$(-2)^2=4$

continuous on $(-\infty,3)\cup(3,\infty)$

$\left.\begin{matrix}(3)^2=9 \\ 8=8\end{matrix}\right\}$ non-removable discontinuity @ 3

$$f(x)=\begin{cases} \frac{|x-3|}{3-x}, & |x-3|>5 \\ x^2-3, & -2\le x\le 8 \end{cases}$$

$x-3>5 \Rightarrow x>8$
or
$x-3<-5 \quad x<-2$

$$\frac{|x-3|}{3-x}=\begin{cases} \frac{x-3}{3-x}=-1, x>3 & x-3>0 \\ \frac{-(x-3)}{3-x}=1, x<3 & x-3<0 \end{cases}$$

$$f(x)=\begin{cases} \frac{|x-3|}{3-x}, & x<-2 \\ x^2-3, & -2\le x\le 8 \\ \frac{|x-3|}{3-x}, & x>8 \end{cases}$$

$$f(x)=\begin{cases} 1, & x<-2 \\ x^2-3, & -2\le x\le 8 \\ -1, & x>8 \end{cases}$$

non-removable (jump) discontinuity @ 8

continuous on $(-\infty,8)\cup(8,\infty)$

Homework #2 - due next Fri, 8/22:

1.3 #11,17,27-35odd, 39-61odd

1.3 #67-77odd; 87, 88

1.4 #7-17odd, 25-28all, 39-47odd, 57, 59

Quiz Time!