1. $\lim_{x\to3}\frac{\sqrt{x+1}-2}{x-3}\cdot\frac{\sqrt{x+1}+2}{\sqrt{x+1}+2} = \lim_{x\to3}\frac{\cancel{x+1-4}\ 1}{\cancel{(x-3)}(\sqrt{x+1}+2)} = \lim_{x\to3}\frac{1}{\sqrt{x+1}+2} = \frac{1}{\sqrt{4}+2} = \boxed{\frac{1}{4}}$

2. $\lim_{x\to2}\frac{2-x}{x^2-4} = \lim_{x\to2}\frac{-1(\cancel{x-2})}{(\cancel{x-2})(x+2)} = \boxed{\frac{-1}{4}}$

3. $\lim_{x\to5}\frac{|x-5|}{5-x} = \lim_{x\to5}\begin{cases}\frac{x-5}{5-x}=-1, & x-5>0,\ x>5\\ \frac{-(x-5)}{5-x}=1, & x-5<0,\ x<5\end{cases}$ does not exis

4. $\lim_{x\to3}\frac{\sqrt{x+1}}{x-4} = \frac{\sqrt{3+1}}{3-4} = \frac{\sqrt{4}}{-1} = \boxed{-2}$

5. $\lim_{x\to0}\frac{\sin x}{x} = \boxed{1}$

6. $\lim_{x\to0}\frac{1-\cos x}{x} = \boxed{0}$

7. $\lim_{x\to3} f(x)$
does not exist

8. $\lim_{x\to-3^-} f(x)$
6

9. $\lim_{x\to1^+} f(x)$
-2

10. $\lim_{x\to-3^+} f(x)$
-1

11. $\lim_{x\to1^-} f(x)$
-5

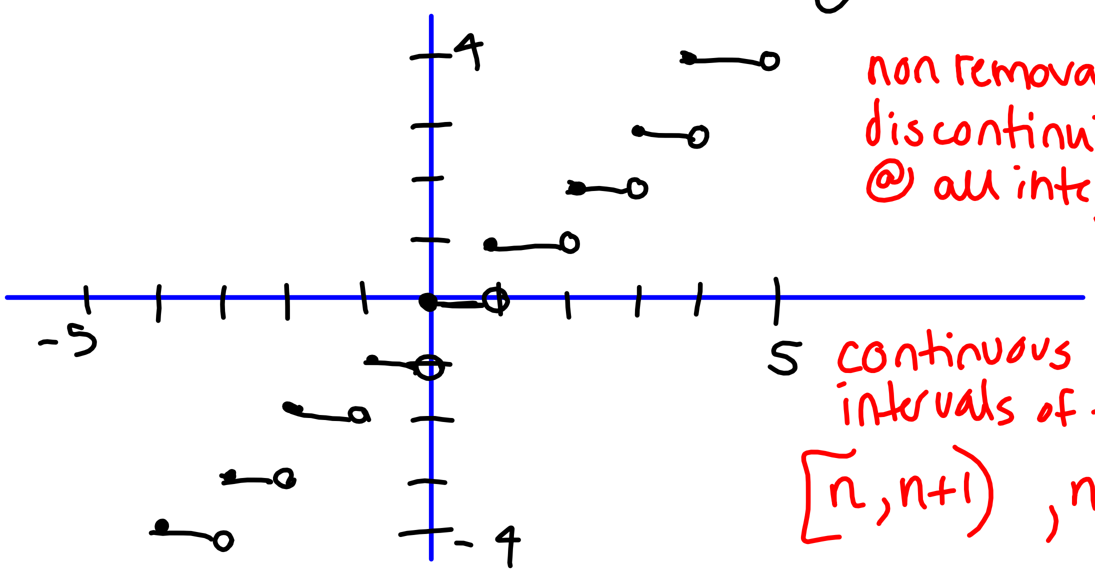
Bonus: State the epsilon-delta definition of the statement $\lim_{x\to c} f(x) = L$.

Given (any) $\varepsilon>0$, there exists a $\delta>0$ such that whenever $|x-c|<\delta$, we have $|f(x)-L|<\varepsilon$.

(if $|x-c|<\delta$, then $|f(x)-L|<\varepsilon$.)

The Greatest Integer Function

$[\![x]\!]$ = the greatest integer less than or equal to x



non removable discontinuities @ all integers

continuous on intervals of the form $[n, n+1)$, $n\in\mathbb{Z}$

22. $\lim_{x\to 2^+} 2x - [\![x]\!]$

$= \lim_{x\to 2^+}(2x) - \lim_{x\to 2^+}[\![x]\!]$

$= 4 - 2$

$= \boxed{2}$

24. $\lim_{x\to 1}\left(1 - [\![-\frac{x}{2}]\!]\right)$

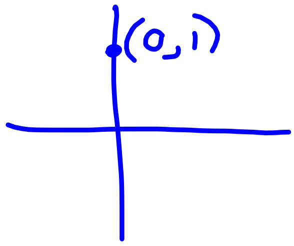
$= \lim_{x\to 1} 1 - \lim_{x\to 1}[\![-\frac{x}{2}]\!]$

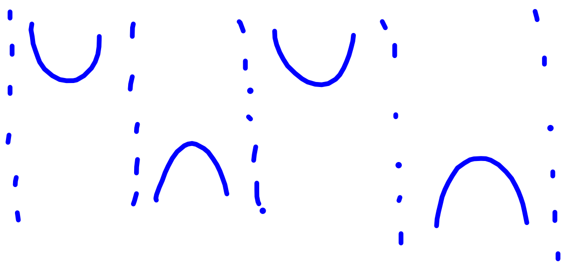
$= 1 - (-1)$

$= \boxed{2}$

20. $\lim_{x\to\frac{\pi}{2}} \sec x = \lim_{x\to\frac{\pi}{2}}\left(\frac{1}{\cos x}\right)$

$\boxed{\text{does not exist}}$

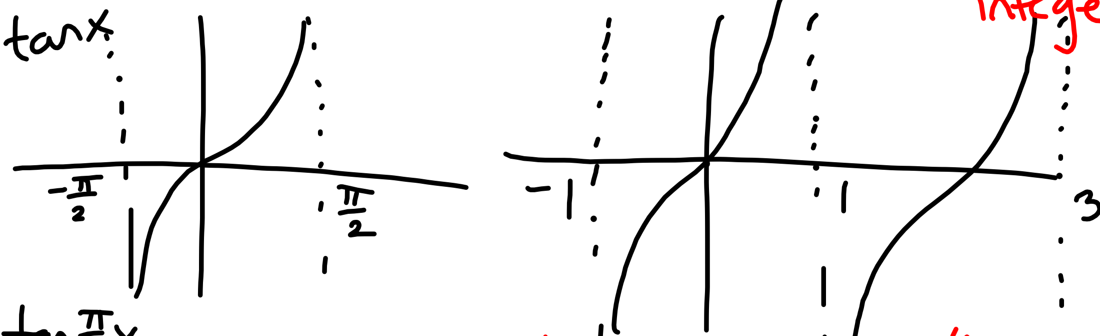




52. $f(x)=\tan\frac{\pi x}{2}$

discuss the (dis)continuity

non-removable discontinuities (asymptotes) @ every odd integer



$\tan\frac{\pi}{2}x$

period: $\frac{\pi}{\pi/2}=2$

continuous on intervals of the form $(2n-1, 2n+1)$, $n\in\mathbb{Z}$

62. $f(x)=\frac{1}{\sqrt{x}}$, $g(x)=x-1$

Discuss the continuity of $f(g(x))$.

$(f\circ g)(x)=\frac{1}{\sqrt{x-1}}$

domain: $\{x \mid x-1>0\}$

$x>1$

$(1,\infty)$

$\boxed{f\circ g \text{ is continuous on its domain } (1,\infty)}$

64. $f(x)=\sin x$; $g(x)=x^2$

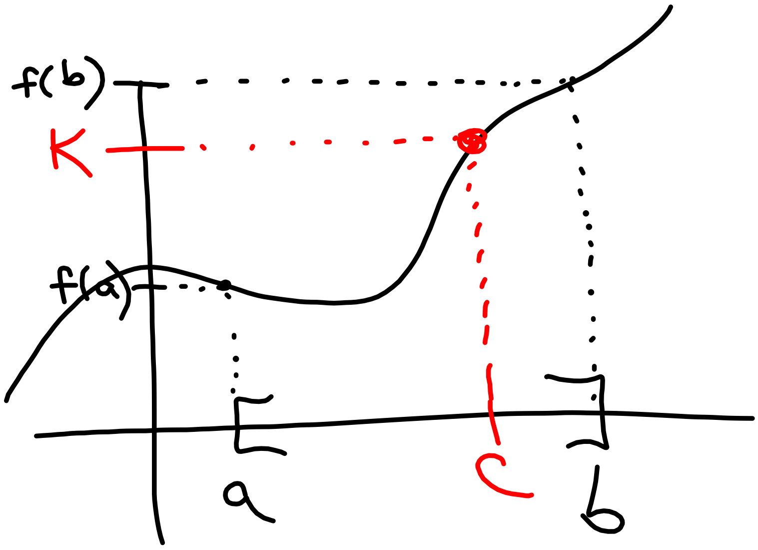
discuss the continuity of $f(g(x))$

$(f\circ g)(x)=\sin(x^2)$

$f\circ g$ is continuous on its domain $(-\infty,\infty)$

Intermediate Value Theorem

If *f* is continuous on the closed interval [*a*,*b*] and *k* is any number between *f(a)* and *f(b)*, then there is at least one number *c* in [*a*,*b*] such that *f(c)=k*.



Does the IVT guarantee a zero in the given interval?

76. $f(x) = x^3 + 3x - 2$, $[0, 1]$

$f(0) = -2 < 0$

$f(1) = 1 + 3 - 2 = 2 > 0$ $\Rightarrow$

Yes, the Intermediate Value Theorem guarantees some $c \in [0,1]$ such that $f(c) = 0$.

84. $f(x)=x^2-6x+8$; $[0,3]$. $f(c)=0$

$f(0)=8 > 0$

$f(3)=9-18+8=-1 < 0$

yes, IVT guarantees such a c.

$f(c)=0$

$c^2-6c+8=0$

$(c-4)(c-2)=0$

$\cancel{c=4}$, $\boxed{c=2}$

86. $f(x)=\dfrac{x^2+x}{x-1}$, $\left[\frac{5}{2},4\right]$, $f(c)=6$

$f\left(\frac{5}{2}\right)=\dfrac{\left(\frac{5}{2}\right)^2+\frac{5}{2}}{\frac{5}{2}-1}=\dfrac{\frac{25}{4}+\frac{10}{4}}{\frac{5}{2}-\frac{2}{2}}=\dfrac{\frac{35}{4}}{\frac{3}{2}}=\frac{35}{4}\cdot\frac{2}{3}=\frac{35}{6}$ <6

$f(4)=\dfrac{4^2+4}{4-1}=\dfrac{16+4}{3}=\dfrac{20}{3}>6$

IVT guarantees such a c

$f(c)=6$

$\dfrac{c^2+c}{c-1}=6$

$c^2+c=6c-6$

$c^2-5c+6=0$

$(c-3)(c-2)=0$

$\boxed{c=3}$, $\cancel{c=2}$

1.5

Infinite Limits

$$\lim_{x \to c} f(x) = \pm\infty$$

means the function increases or decreases without bound; i.e. the graph of the function approaches a vertical asymptote

Finding Vertical Asymptotes

x-values at which a function is undefined result in either holes in the graph or vertical asymptotes. Holes result when a function can be rewritten so that the factor which yields the discontinuity cancels. Factors that can't cancel yield vertical asymptotes.

Examples:

$f(x) = \frac{1}{x(x+3)}$ has vertical asymptotes at $x = 0$ and $x = 3$

$f(x) = \frac{(x+2)(x+3)}{x(x+3)}$ has a vertical asymptote at $x = 0$ and a hole at $x = -3$

Rules involving infinite limits

Let $\lim_{x \to c} f(x) = \infty \ and \ \lim_{x \to c} g(x) = L$

1. $\lim_{x \to c} [f(x) \pm g(x)] = \infty$

2. $\lim_{x \to c} [f(x)g(x)] = \begin{cases} \infty, & L > 0 \\ -\infty, & L < 0 \end{cases}$

3. $\lim_{x \to c} \frac{g(x)}{f(x)} = 0$

Find the vertical asymptotes (if any).

14. $f(x) = \dfrac{-4x}{x^2+4}$ none

24. $h(x) = \dfrac{\overbrace{x^2-4}^{(x-2)\cancel{(x+2)}}}{x^3+2x^2+x+2}$ none

$x^2(x+2)+1(x+2)$

$(x^2+1)\cancel{(x+2)}$

28. $g(\theta) = \dfrac{\tan\theta}{\theta}$ $x=0$, & all odd multiples of $\pi/2$

42. $\lim\limits_{x\to 0^-}\left(x^2-\dfrac{1}{x}\right) = \lim\limits_{x\to 0^-}\left(\dfrac{x^3-1}{x}\right)$

46. $\lim\limits_{x\to 0}\dfrac{x+2}{\cot x}$

Homework for Test #1:

HW#1 (submitted Fri. 8/15)

- 1.2 #1-7odd,9-18all
- 1.2 #23, 25, 27, 29, 30, 31 epsilon-delta

HW#2 (due ~~Fri.~~ Mon 8/~~22~~ 25)

- 1.3 #11,17,27-35odd, #39-61odd (<-- not listed on your syllabus!)
- 1.3 #67-77odd; 87, 88 (<-- not listed on your syllabus!)
- 1.4 #7-17odd; 25-28all; 39-47odd; 57, 59
- **1.4 #19,21,23,51,63,69,71,83,85**
- **1.5 #9-17odd; 29-47odd; 53-56all**

HW #3 (due test day)

- Test #1 Practice Problems
- Ch 1 review pp. 88-89
- (*recommended* - Old Test #1 on web; *solutions can be found in course notes from last term*)

HW #4 (not due until after the test, but will still help you with limits that will be on the test)

- 2.1 (derivative definition) - p.101-102 #1-23odd

Quiz #2 - when?

Test #1 - on syllabus for Friday, 8/29; earlier?