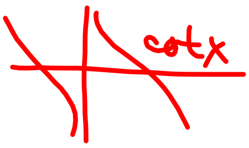
42. $\lim_{x\to 0^-}\left(x^2-\frac{1}{x}\right) = \lim_{x\to 0^-}\left(\frac{x^3-1}{x}\right) = \lim_{x\to 0^-}\frac{(x-1)(x^2+x+1)}{x}$

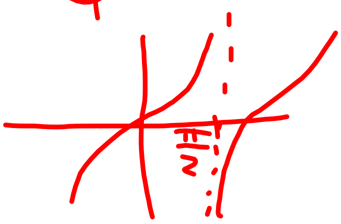
$= \boxed{+\infty}$

46. $\lim_{x\to 0}\frac{x+2}{\cot x} = \lim_{x\to 0}(x+2)(\tan x) = \boxed{0}$



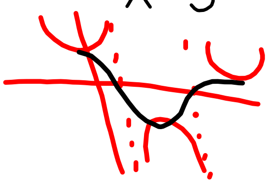
48. $\lim_{x\to \frac{1}{2}} x^2\tan \pi x = \left(\lim_{x\to\frac{1}{2}} x^2\right)\left(\lim_{x\to\frac{1}{2}} \tan\pi x\right)$

$= \left(\frac{1}{2}\right)^2 \cdot \lim_{x\to\frac{1}{2}} \tan\pi x$



$\boxed{\text{does not exist}}$

52. $\lim_{x\to 3^+}\sec\frac{\pi x}{6} = \boxed{-\infty}$

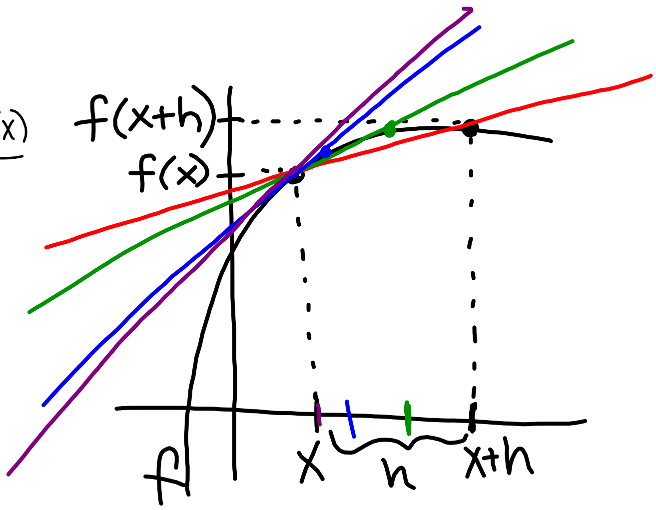


2.1 The Derivative & The Tangent Line Problem

secant line
crosses through a function at two points

slope of the secant line:

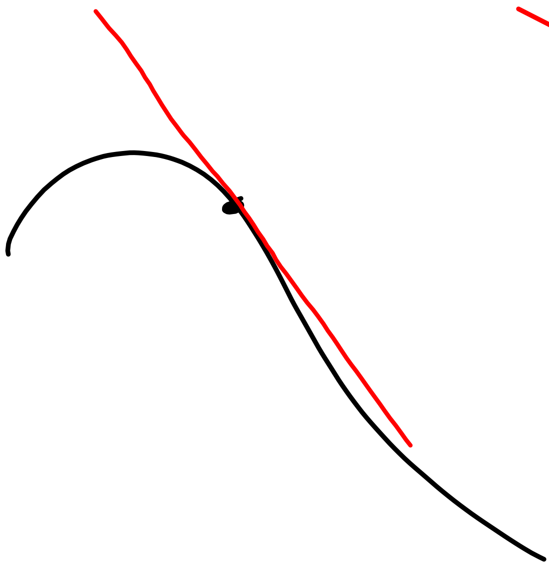
$$\frac{f(x+h)-f(x)}{x+h-x} = \frac{f(x+h)-f(x)}{h}$$

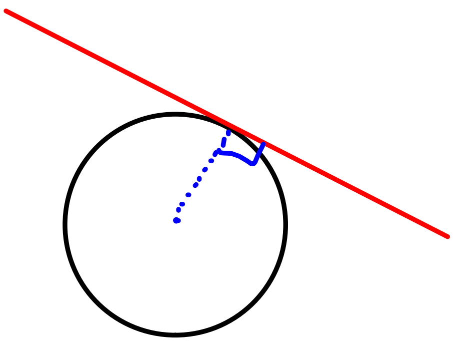


what happens as $h \to 0$?

$$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

As $h \to 0$, the secant line approximates the tangent line, and the limit is the slope of the tangent line and we call it **the derivative of f at x**.





$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$\boldsymbol{f'(x)}$	"f prime of x"
$\boldsymbol{\frac{dy}{dx}}$	"derivative of y with respect to x"
$\boldsymbol{y'}$	"y prime"
$\boldsymbol{\frac{d}{dx}[f(x)]}$	"the derivative with respect to x of $f(x)$"
$\boldsymbol{D_x[y]}$	"the partial derivative with respect to x of y"

The Derivative

The slope of the tangent line to the graph of f at the point $(c, f(c))$ is given by:

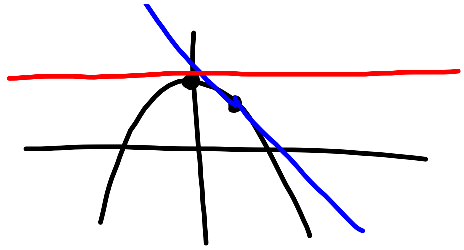
$$m = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x}$$

The derivative of f at x is given by

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

8. $g(x) = 5 - x^2$

find slope of tangent line at the points $(2,1)$ & $(0,5)$



$(c, g(c))$

$m =$

$$g'(c) = \lim_{h\to 0} \frac{g(c+h) - g(c)}{h} = \lim_{h\to 0} \frac{5-(2+h)^2 - 1}{h}$$

$$= \lim_{h\to 0} \frac{5-(4+4h+h^2)-1}{h} = \lim_{h\to 0} \frac{-4h-h^2}{h} = \lim_{h\to 0} \frac{\cancel{h}(-4-h)}{\cancel{h}}$$

$$= \boxed{-4}$$

$$g'(0) = \lim_{h\to 0} \frac{5-(0+h)^2-5}{h} = \lim_{h\to 0} \frac{-h^2}{h} = \lim_{h\to 0} -h = \boxed{0}$$

20. $f(x) = x^3 + x^2$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

find the derivative

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^3 + (x+h)^2 - (x^3+x^2)}{h}$$

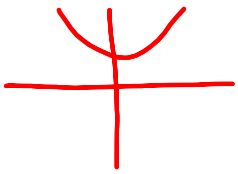
$$= \lim_{h\to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 + \cancel{x^2} + 2xh + h^2 - \cancel{x^3} - \cancel{x^2}}{h}$$

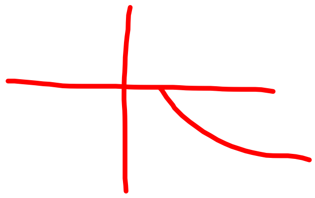
$$= \lim_{h\to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2 + 2x + h)}{\cancel{h}}$$

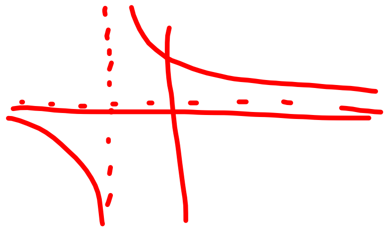
$$= 3x^2 + 3x(0) + 0^2 + 2x + 0 = \boxed{3x^2 + 2x}$$

1. Find the domain and range of the function.

a. $y = x^2 + 1$ b. $y = -\sqrt{x-2}$ c. $y = \frac{1}{x+2}$







D: $(-\infty, \infty)$; $[2, \infty)$; $(-\infty, -2) \cup (-2, \infty)$

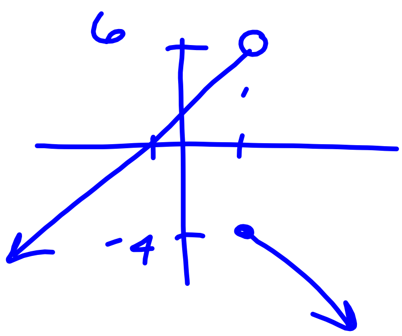
R: $[1, \infty)$; $(-\infty, 0]$; $(-\infty, 0) \cup (0, \infty)$

2. Determine the domain and range of the piecewise function, and evaluate the function as indicated.

$$f(x) = \begin{cases} 3x + 3, & x < 1 \\ -x^2 - 3, & x \geq 1 \end{cases}$$

a. domain b. range c. $f(-2)$ d. $f(3)$ e. $f(t^2 + 5)$

$(-\infty, \infty)$ $(-\infty, 6)$ -3 -12 $-(t^2+5)^2 - 3$



5. Find the limit L, then use the $\varepsilon - \delta$ definition to prove that the limit is L.

$\lim_{x\to 2}(2x-3) = 2(2)-3 = 1$ $\boxed{L=1}$

$|2x-3-1| = |2x-4| = 2|x-2| < \varepsilon$ $C=2$

$|x-2| < \varepsilon/2$

Proof:

Given $\varepsilon > 0$, take $\boxed{\delta = \varepsilon/2}$.

Then whenever $|x-2| < \delta$, we have that $|(2x-3)-1| = |2x-4| = 2|x-2| < 2\frac{\varepsilon}{2} = \varepsilon$,

i.e. $|x-c| < \delta$ implies $|f(x)-L| < \varepsilon$.

Hence $\lim_{x\to 2}(2x-3)$ is indeed 1.

6. Find the limit.

a. $\lim_{x\to 2}(2x^3 - x + 5)$ $= \boxed{19}$

b. $\lim_{x\to 2}\frac{x+4}{x^2+1}$ $\boxed{\frac{6}{5}}$

c. $\lim_{x\to -2}\cos \pi x$ $\boxed{1}$

7. Use the given information to evaluate the given limits.

$\lim_{x\to c} f(x) = -3$, $\lim_{x\to c} g(x) = 5$

a. $\lim_{x\to c}\left[2f(x) + \sqrt{g(x)}\right]$

$= 2(-3) + \sqrt{5}$

$= \boxed{-6 + \sqrt{5}}$

b. $\lim_{x\to c}\left[3f(x)\sqrt{g(x)}\right]$

$3(-3)\sqrt{5}$

$\boxed{-9\sqrt{5}}$

8. Find the limit (if it exists).

$$\lim_{x\to -1}\frac{x^2 - x - 2}{x^2 - 1} = \lim_{x\to -1}\frac{(x-2)\cancel{(x+1)}}{(x-1)\cancel{(x+1)}} = \boxed{\frac{3}{2}}$$

9. Find the limit (if it exists).

$$\lim_{x\to -1}\frac{x^2 - 9}{x^2 - 5x + 6} = \lim_{x\to -1}\frac{(x+3)\cancel{(x-3)}}{(x-2)\cancel{(x-3)}} = \boxed{-\frac{2}{3}}$$

10. Find the limit (if it exists).

$$\lim_{x\to 1}\frac{\sqrt{x+3}-2}{x^2-1}\cdot\frac{\sqrt{x+3}+2}{\sqrt{x+3}+2}=\lim_{x\to 1}\frac{x+3-4}{(x^2-1)(\sqrt{x+3}+2)}$$

$$=\lim_{x\to 1}\frac{\cancel{x-1}\ 1}{(\cancel{x-1})(x+1)(\sqrt{x+3}+2)}=\frac{1}{2(\sqrt{4}+2)}=\boxed{\frac{1}{8}}$$

11. Determine the limit of the trigonometric function (if it exists).

$$\lim_{x\to 0}\frac{\sin 5x}{2x}=\lim_{x\to 0}\frac{\sin 5x}{5x}\cdot\frac{5}{2}$$

$$=\left(\lim_{x\to 0}\frac{\sin 5x}{5x}\right)\left(\lim_{x\to 0}\frac{5}{2}\right)=1\cdot\frac{5}{2}=\boxed{\frac{5}{2}}$$

Homework for Test #1:

HW#1 (submitted Fri. 8/15)
- 1.2 #1-7odd,9-18all
- 1.2 #23, 25, 27, 29, 30, 31 epsilon-delta

HW#2 (due ~~Fri.~~ Mon 8/~~22~~ 25)
- 1.3 #11,17,27-35odd, #39-61odd (<-- not listed on your syllabus!)
- 1.3 #67-77odd; 87, 88 (<-- not listed on your syllabus!)
- 1.4 #7-17odd; 25-28all; 39-47odd; 57, 59
- **1.4 #19,21,23,51,63,69,71,83,85**
- **1.5 #9-17odd; 29-47odd; 53-56all**

HW #3 (due test day)
- Test #1 Practice Problems
- Ch 1 review pp. 88-89
- (*recommended* - Old Test #1 on web; *solutions can be found in course notes from last term*)

HW #4 (not due until after the test, but will still help you with limits that will be on the test)
- 2.1 (derivative definition) - p.101-102 #1-23odd

Quiz #2 - when?

Test #1 - on syllabus for Friday, 8/29; earlier?

Test Wed