Turn in HW#2

- 1.3 #11,17,27-35odd, #39-61odd
- 1.3 #67-77odd; 87, 88
- 1.4 #7-17odd; 25-28all; 39-47odd; 57, 59
- 1.4 #19,21,23,51,63,69,71,83,85
- 1.5 #9-17odd; 29-47odd; 53-56all

12. Determine the limit of the trigonometric function (if it exists).

$$\lim_{x\to 0}\frac{\tan^2 x}{2x} = \lim_{x\to 0}\frac{\sin^2 x}{\cos^2 x\cdot 2x} = \lim_{x\to 0}\underbrace{\frac{\sin x}{x}}_{\downarrow\ 1}\cdot\underbrace{\frac{\sin x}{2\cos^2 x}}_{\downarrow\ 0} = 1\cdot 0 = \boxed{0}$$

13. Determine the limit of the trigonometric function (if it exists).

$$\lim_{x\to 0}\frac{3(1-\cos x)^2}{x} = \lim_{x\to 0}\underbrace{\frac{1-\cos x}{x}}_{\downarrow\ 0}\cdot\underbrace{3(1-\cos x)}_{\downarrow\ 0} = \boxed{0}$$

14. Find

$$\lim_{\Delta x \to 0} \frac{f(x+h) - f(x)}{h}, \quad \text{where } f(x) = x + 1$$

$=1$

15. Find

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}, \quad \text{where } f(x) = 2x^2 - 1$$

$$\lim_{h \to 0} \frac{2(x+h)^2 - 1 - (2x^2 - 1)}{h} = \ldots = 4x$$

16. Use the Squeeze Theorem to find $\lim_{x \to 0} f(x)$.

$$f(x) = x^2 \sin \frac{3}{x}$$

If $f(x) \le g(x) \le h(x)$

& $\lim_{x \to c} f(x) = L = \lim_{x \to c} h(x)$,

then $\lim_{x \to c} g(x) = L$

$$-1 \le \sin \frac{3}{x} \le 1$$

$$-x^2 \le x^2 \sin \frac{3}{x} \le x^2$$

$$\underbrace{\lim_{x \to 0} (-x^2)}_{=0} \le \lim_{x \to 0} \left(x^2 \sin \frac{3}{x}\right) \le \underbrace{\lim_{x \to 0} x^2}_{=0}$$

By the Squeeze Thm, $\lim_{x \to 0} \left(x^2 \sin \frac{3}{x}\right) = \boxed{0}$

17. Use the Squeeze Theorem to find $\lim_{x \to 0} f(x)$.

$$2 - 3x^2 \le f(x) \le 2 + 5x^2$$

$$\underbrace{\lim_{x \to 0} (2 - 3x^2)}_{=2} \le \lim_{x \to 0} f(x) \le \underbrace{\lim_{x \to 0} (2 + 5x^2)}_{=2}$$

By the Squeeze Thm, $\lim_{x \to 0} f(x) = \boxed{2}$

18. Find the limit (if it exists).

$$\lim_{x\to 4^+} \frac{|x-4|}{x-4}$$

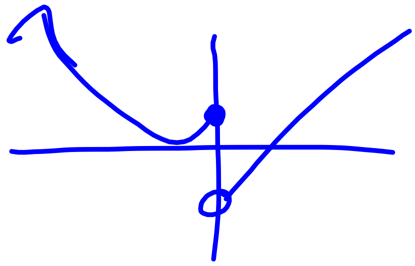
$= \boxed{1}$

$$\frac{|x-4|}{x-4} = \begin{cases} \frac{x-4}{x-4} = 1, & x-4>0,\ x>4 \\ \frac{-(x-4)}{x-4} = -1, & x-4<0,\ x<4 \end{cases}$$

19. Find the limit (if it exists).

$$\lim_{x\to 0^+} f(x), \quad f(x) = \begin{cases} 2x^2+2x+1, & x \le 0 \\ x-3, & x>0 \end{cases}$$

$= \boxed{-3}$



20. Find the limit (if it exists).

$$\lim_{x\to 2} f(x), \quad f(x) = \begin{cases} 10-x, & x \le 2 \\ x^2+2x, & x>2 \end{cases}$$

$10-2=8$

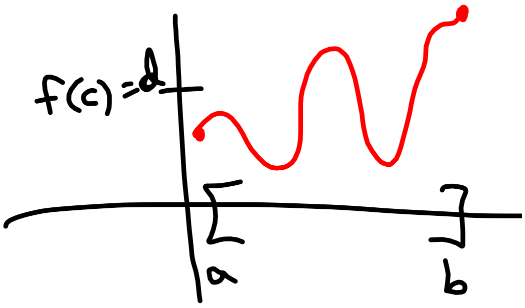
$2^2+2(2)=8$

$\lim_{x\to 2} f(x) = \boxed{8}$

8. Determine if the Intermediate Value Theorem guarantees a c in the interval $[-2, 3]$ such that $f(c) = -4$, and if so, find all such values of c.

$f(x) = x^2 - 7x + 2$

$f(-2) = 20 > -4$
$f(3) = -10 < -4$
IVT applies



$x^2-7x+2=-4$

$x^2-7x+6=0$

$(x-6)(x-1)=0$

$\cancel{x=6}$, $\boxed{x=1}$

9. Discuss the continuity of the function (identify all discontinuities, if any, as removable or non-removable).

$$f(x) = \frac{x^2 - 7x + 10}{x^2 - 3x + 2} = \frac{(x-5)(x-2)}{(x-2)(x-1)}$$

removable discontinuity @ $x=2$

non-removable discontinuity @ $x=1$

continuous on $(-\infty, 1) \cup (1, 2) \cup (2, \infty)$

The Derivative

The slope of the tangent line to the graph of f at the point $(c, f(c))$ is given by:

$$m = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x}$$

The derivative of f at x is given by

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

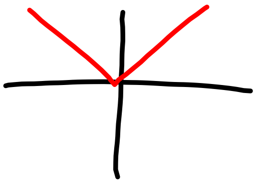
2.1 Differentiability & Continuity

Alternative definition of the derivative at the point $(c, f(c))$:

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

All differentiable functions are continuous, but
not all continuous functions are differentiable.

e.g. $f(x) = |x|$



$$\lim_{x \to 0} \frac{|x| - |0|}{x - 0} = \lim_{x \to 0} \frac{|x|}{x}$$

|x| is not differentiable, as left & right-hand limits are different!

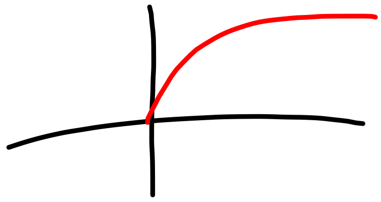
$$\lim_{x \to 0^-} \frac{|x|}{x} = \lim_{x \to 0^-} \frac{-x}{x} = -1 \ ; \ \lim_{x \to 0^+} \frac{|x|}{x} = \lim_{x \to 0^+} \frac{x}{x} = 1$$

$f(x) = |x + 3|$ is not differentiable @ $x = -3$

$$\lim_{x \to -3^-} \frac{|x+3| - |-3+3|}{x - (-3)} = -1$$

$$\lim_{x \to -3^+} \frac{|x+3| - |-3+3|}{x - (-3)} = 1$$

$f(x) = \sqrt{x}$



$$\lim_{x \to 0^+} \frac{\sqrt{x} - \sqrt{0}}{x - 0}$$

$$= \lim_{x \to 0^+} \frac{\sqrt{x}}{x} = \lim_{x \to 0^+} \frac{x^{1/2}}{x^1} = \lim_{x \to 0^+} \frac{1}{x^{1/2}}$$

$$= \lim_{x \to 0^+} \frac{1}{\sqrt{x}} = \infty$$ $\Rightarrow$ vertical tangent line

$\Rightarrow$ f is not differentiable @ $x = 0$

2.2 Basic Differentiation Rules

$$\frac{d}{dx}[f(x)] = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

1. The derivative of a constant function is zero, i.e.,

for $c \in \mathbb{R}$, $\frac{d}{dx}[c] = 0$

Proof:

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{c - c}{h} = \lim_{h \to 0} 0$$

$$= \boxed{0}$$

2. Power Rule for $n \epsilon \mathbb{Q}$, $\frac{d}{dx}[x^n] = nx^{n-1}$

Proof:

Recall the binomial expansion:

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2}a^{n-2}b^2 + \cdots + \frac{n!}{k!\,(n-k)!}a^{n-k}b^k + \cdots + b^n$$

$$\lim_{h\to 0}\frac{(x+h)^n - x^n}{h} = \lim_{h\to 0}\frac{\cancel{x^n} + nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 + \ldots + h^n - \cancel{x^n}}{h}$$

$$= \lim_{h\to 0}\frac{\cancel{h}\left(nx^{n-1} + \frac{n(n-1)}{2}x^{n-2}h + \ldots + h^{n-1}\right)}{\cancel{h}} \to 0$$

$$= \boxed{nx^{n-1}}$$

$\frac{d}{dx}x^1 = 1 \cdot x^0$

Special case: $\frac{d}{dx}[x] = 1$

Examples:

$\frac{d}{dx}[x^7] = 7x^6$

$\frac{d}{dx}[\pi^3] = 0$

$\frac{d}{dx}[2e] = 0$

$\frac{d}{dx}[\sqrt{x}] = \frac{d}{dx}[x^{1/2}] = \boxed{\frac{1}{2}x^{-1/2}} = \frac{1}{2\sqrt{x}}$

$\frac{d}{dx}\left[\frac{1}{x^3}\right] = \frac{d}{dx}[x^{-3}] = \boxed{-3x^{-4}} = \frac{-3}{x^4}$

Homework for Test #1:

HW#1 (submitted Fri. 8/15)
- 1.2 #1-7odd,9-18all
- 1.2 #23, 25, 27, 29, 30, 31 epsilon-delta

HW#2 (submitted Mon. 8/25)
- 1.3 #11,17,27-35odd, #39-61odd (<-- not listed on your syllabus!)
- 1.3 #67-77odd; 87, 88 (<-- not listed on your syllabus!)
- 1.4 #7-17odd; 25-28all; 39-47odd; 57, 59
- 1.4 #19,21,23,51,63,69,71,83,85
- 1.5 #9-17odd; 29-47odd; 53-56all

HW #3 (due ~~test day, Wed. 8/27)~~ Fri

- Test #1 Practice Problems
- Ch 1 review pp. 88-89 even #'s
- (*recommended* - Old Test #1 on web; *solutions can be found in course notes from last term*)

~~HW #4 (not due until after the test, but will still help you with limits that will be on the test)~~
- 2.1 (derivative definition) - p.101-102 #1-23odd

Quiz #2 - when?

Test #1 - ~~Wed 8/27~~ Fri 8/29