2.2 Basic Differentiation Rules

$$\frac{\Delta y}{\Delta x}$$

1. The derivative of a constant function is zero, i.e.,

for $c \in \mathbb{R}$, $\frac{d}{dx}[c] = 0$

2. Power Rule for $n \epsilon \mathbb{Q}$, $\frac{d}{dx}[x^n] = nx^{n-1}$

3. Constant Multiple Rule $\in \mathbb{R}$, $\frac{d}{dx}[cf(x)] = cf'(x)$

4. Sum & Difference Rules $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

$$\frac{d}{dx}[c \cdot f(x)] = [c \cdot f(x)]'$$

$$= \lim_{h \to 0} \frac{c \cdot f(x+h) - c \cdot f(x)}{h}$$

$$= \lim_{h \to 0} c \cdot \frac{f(x+h - f(x)}{h}$$

$$= c \cdot \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = c \cdot \frac{d}{dx}[f(x)]$$

Examples:

$f(x) = 3x^2$

$f'(x) = 3 \cdot (x^2)' = 3(2x) = \boxed{6x}$

$f(x) = \frac{3}{x} = 3x^{-1}$

$f'(x) = \boxed{-3x^{-2}} = \frac{-3}{x^2}$

$\frac{d}{dx}[x^1] = 1x^0$
$= 1$

$g(x) = 2x^3 - x^2 + 3x$

$g'(x) = \boxed{6x^2 - 2x + 3}$

$y = 4x^{3/2} - 5x^4 + 2x^{\frac{1}{3}} - 7$

$y' = \boxed{6x^{1/2} - 20x^3 + \frac{2}{3}x^{-2/3}}$

Derivatives of Trig Functions

1. $\frac{d}{dx}[\sin x] = \cos x$
2. $\frac{d}{dx}[\cos x] = -\sin x$
3. $\frac{d}{dx}[\tan x] = \sec^2 x$
4. $\frac{d}{dx}[\cot x] = -\csc^2 x$
5. $\frac{d}{dx}[\sec x] = \sec x \tan x$
6. $\frac{d}{dx}[\csc x] = -\csc x \cot x$

Proof that (sin x)'=cos x

$$[\sin x]' = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h} =$$

$$= \lim_{h\to 0} \frac{\sin x\cos h + \cos x \sin h - \sin x}{h} =$$

$$= \lim_{h\to 0} \left[\frac{-\sin x + \sin x \cos h}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h\to 0} \left[-\sin x \left(\frac{1-\cos h}{h} \right) \right] + \lim_{h\to 0} \left[\cos x \left(\frac{\sin h}{h} \right) \right]$$

$$= (-\sin x)(0) + (\cos x)(1) = \boxed{\cos x}$$

$$[\csc x]' = -\csc x \cot x = -\frac{1}{\sin x} \cdot \frac{\cos x}{\sin x}$$

$$= \lim_{h\to 0} \frac{\csc(x+h) - \csc x}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{\sin(x+h)} - \frac{1}{\sin x}}{h} =$$

$$= \lim_{h\to 0} \frac{\frac{1}{\sin x\cos h + \cos x \sin h} - \frac{1}{\sin x}}{h} =$$

$$= \lim_{h\to 0} \frac{\sin x - (\sin x \cos h + \cos x \sin h)}{h \sin x (\sin x \cos h + \cos x \sin h)}$$

$$= \lim_{h\to 0} \frac{\sin x (1 - \cos h) - \cos x \sin h}{h \sin x (\sin x \cos h + \cos x \sin h)}$$

$$= \lim_{h\to 0} \left(\frac{1-\cos h}{h} \cdot \frac{\cancel{\sin x}\ 1}{\cancel{\sin x}(\sin x \underbrace{\cos h}_{1} + \cos x \underbrace{\sin h}_{0})} - \frac{\sin h}{h} \cdot \frac{\cos x}{\sin x (\sin x \cos h + \cos x \sin h)} \right)$$

$$= \cancel{0 \cdot \frac{1}{\sin x}} - 1 \cdot \frac{\cos x}{\sin x (\sin x \cdot 1 + \cancel{\cos x \cdot 0})}$$

$$= \frac{-\cos x}{\sin x \sin x} = \boxed{-\csc x \cot x}$$

The Derivative

The slope of the tangent line to the graph of f at the point $(c, f(c))$ is given by:

$$m = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c+\Delta x) - f(c)}{\Delta x}$$

The derivative of f at x is given by

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

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2.2

22. $y = 5 + \sin x$

$y' = \boxed{\cos x}$

24. $y = \frac{5}{(2x)^3} + 2\cos x = \frac{5}{8}\left(x^{-3}\right) + 2\cos x$

$y' = \boxed{\frac{-15}{8}x^{-4} - 2\sin x}$

44. $h(x) = \frac{2x^3 - 3x + 1}{x} = \frac{2x^3}{x} - \frac{3x}{x} + \frac{1}{x} = 2x^2 - 3 + x^{-1}$

$h'(x) = \boxed{4x - x^{-2}} = 4x - \frac{1}{x^2} = \frac{4x^3 - 1}{x^2}$

46. $y = 3x(6x - 5x^2)$

$y' =$

52. $f(x) = \frac{2}{\sqrt[3]{x}} + 3\cos x$

HW #3 (due test day, Fri. 8/29)

- **Test #1 Practice Problems**
- **Ch 1 review pp. 88-89 even problems only**
- (*recommended* - Old Test #1 on web; *solutions can be found in course notes from last term*)
- **2.1 #1-23odd -** Find the derivative by the limit process
  #29-32 all - find the equation of the tangent line
  #61-69 odd - Use the alternate form to find the derivative
  #71-79 odd - Describe the x-values where the function is differentiable (given a graph)
- **2.2 #3-51 odd** - Find the derivative using the basic derivative rules
  #91-94 all; 101, 102 - use the derivative to solve rate of change word problems

Test #1 - Fri 8/29

Recommended: Work through intuitive exercises on **Khan Academy**:

- Slope of secant lines
- Tangent lope is limiting value of secant slope
- Derivative intuition
- Visualizing derivatives
- Graphs of functions and their derivatives
- The formal and alternate form of the derivative
- Derivatives 1
- Recognizing slopes of curves
- Power rule
- Special derivatives