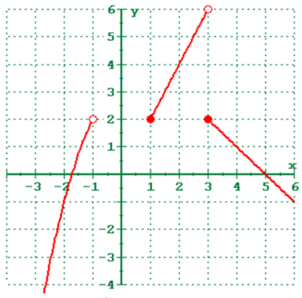
<u>goals for today</u>:

- find equation of tangent line
- confirm limit definition of the derivative
- determine where/if a function is differentiable (or not) given graph or using alternate form of limit definition
- rate of change word problems
- last minute review questions

	As $x \to$	$f(x) \to$
1.	-1^-	2
2.	3^+	2
3.	3^-	6
4.	1^+	2
5.	$-\infty$	$-\infty$



8. $\lim_{x\to 5^-} \frac{|x-5|}{5-x} = \boxed{1}$

$\frac{|x-5|}{5-x} = \begin{cases} \frac{x-5}{5-x} = -1, & x-5>0,\ x>5 \\ \frac{-(x-5)}{5-x} = 1, & x-5<0,\ x<5 \end{cases}$

9. $\lim_{x\to 4} \frac{\sqrt{x+5}-3}{x-4} =$

$\frac{\sqrt{x+5}-3}{x-4} \cdot \frac{\sqrt{x+5}+3}{\sqrt{x+5}+3} = \frac{x+5-9}{(x-4)(\sqrt{x+5}+3)}$; $\frac{1}{\sqrt{4+5}+3} = \boxed{\frac{1}{6}}$

10. $\lim_{x\to -1} \frac{x^2-1}{x+1} =$

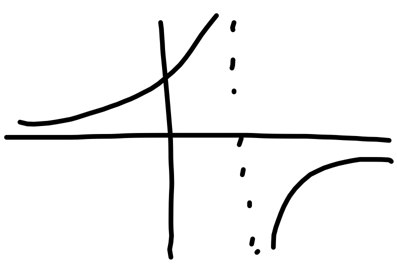
$\lim_{x\to -1} \frac{(x-1)\cancel{(x+1)}}{\cancel{x+1}} = \boxed{-2}$

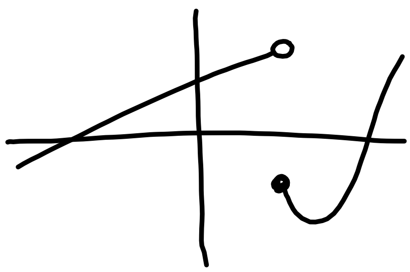
11. $\lim_{x\to -2} 5x+4 = 5(-2)+4 = \boxed{-6}$

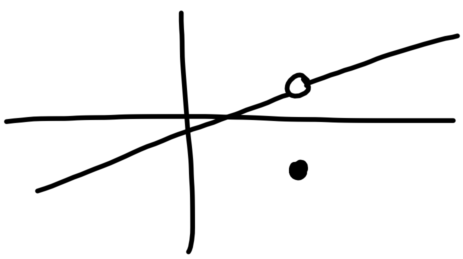
6. A function f is continuous at c if $\lim_{x\to c} f(x) =$ <u>$f(c)$</u>

7. According to the <u>Squeeze Theorem</u> if $f(x) \le g(x) \le h(x)$, and $\lim_{x\to c} f(x) = L = \lim_{x\to c} h(x)$, then $\lim_{x\to c} g(x) =$ <u>L</u>.

Types of discontinuities :







Intermediate Value Theorem:

If a function f is continuous on $[a,b]$, $f(a) < y$ and $f(b) > y$ (or $f(a) > y$ and $f(b) < y$), then there exists at least one $c \in (a,b)$ such that $f(c) = y$.

Does IVT guarantee a $c \in [-5, 10]$ such that $f(c) = 2$ for $f(x) = 3x - 1$?

$$\left.\begin{array}{l} f(-5) = 3(-5) - 1 = -16 < 2 \\ f(10) = 3(10) - 1 = 29 > 2 \end{array}\right\} \text{IVT guarantees a } c$$

$3x - 1 = 2$

$3x = 3$

$\boxed{x = 1}$

The Derivative

The slope of the tangent line to the graph of f at the point $(c, f(c))$ is given by:

$$m = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c+\Delta x) - f(c)}{\Delta x}$$

The derivative of f at x is given by

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

2.2 Basic Differentiation Rules

1. The derivative of a constant function is zero, i.e., for $c \in \mathbb{R}$, $\frac{d}{dx}[c] = 0$

2. Power Rule for $n \epsilon \mathbb{Q}$, $\frac{d}{dx}[x^n] = nx^{n-1}$

3. Constant Multiple Rule $\in \mathbb{R}$, $\frac{d}{dx}[cf(x)] = cf'(x)$

4. Sum & Difference Rules $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

Derivatives of Trig Functions

1. $\frac{d}{dx}[\sin x] = \cos x$
2. $\frac{d}{dx}[\cos x] = -\sin x$
3. $\frac{d}{dx}[\tan x] = \sec^2 x$
4. $\frac{d}{dx}[\cot x] = -\csc^2 x$
5. $\frac{d}{dx}[\sec x] = \sec x \tan x$
6. $\frac{d}{dx}[\csc x] = -\csc x \cot x$

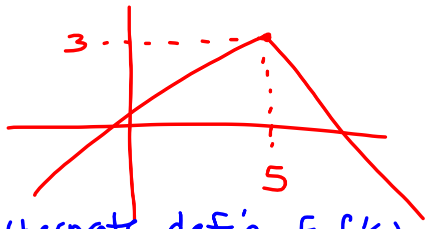
Find the equation of the tangent line to $f(x) = x^3 - x$ at the point $(2, 6)$.

$f'(x) = 3x^2 - 1$

$m = f'(2) = 3(2)^2 - 1 = 11$

$y - y_1 = m(x - x_1)$

$y - 6 = 11(x - 2) \Longrightarrow \boxed{y = 11x - 16}$



$f(x) = -|x-5| + 3$

f is continuous @ $x=5$,

but not differentiable @ $x=5$

Alternate def'n of $f'(c)$

$$\lim_{x \to c} \frac{f(x)-f(c)}{x-c}$$

$$\lim_{x \to 5^-} \frac{-|x-5|+3-3}{x-5} = \lim_{x \to 5^-} \frac{-|x-5|}{x-5} = \lim_{x \to 5^-} \frac{-[-(x-5)]}{x-5} = \boxed{1}$$

$$\lim_{x \to 5^+} \frac{-|x-5|+3-3}{x-5} = \lim_{x \to 5^+} \frac{-|x-5|}{x-5} = \lim_{x \to 5^+} \frac{-(x-5)}{x-5} = \boxed{-1}$$

Because these left- & right- limits are not equal, f is not differentiable.

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

2.2

22. $y = 5 + \sin x$

$y' = \boxed{\cos x}$

24. $y = \frac{5}{(2x)^3} + 2\cos x = \frac{5}{8}(x^{-3}) + 2\cos x$

$y' = \boxed{\frac{-15}{8}x^{-4} - 2\sin x}$

44. $h(x) = \frac{2x^3-3x+1}{x} = \frac{2x^3}{x} - \frac{3x}{x} + \frac{1}{x} = 2x^2 - 3 + x^{-1}$

$h'(x) = \boxed{4x - x^{-2}} = 4x - \frac{1}{x^2} = \frac{4x^3-1}{x^2}$

46. $y = 3x(6x-5x^2) = 18x^2 - 15x^3$

$y' = \boxed{36x - 45x^2}$

52. $f(x) = \frac{2}{\sqrt[3]{x}} + 3\cos x = 2x^{-1/3} + 3\cos x$

$f'(x) = \boxed{\frac{-2}{3}x^{-4/3} - 3\sin x}$

2.2 cont.

$s(t) = $ position

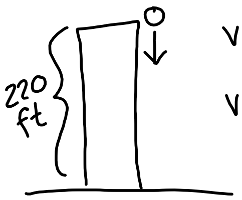
$v(t) = s'(t) = $ velocity

$a(t) = v'(t) = s''(t) = $ acceleration

average velocity: $\dfrac{\Delta s}{\Delta t}$ (slope of secant)

instantaneous velocity $= s'(t)$ (slope of tangent)

92. initial velocity $V_0 = -22$ ft/s



$v(3) = ?$

$v(t) = ?$ after falling 108 ft

$s(t) = \frac{1}{2}gt^2 + v_0 t + s_0$ $\quad g = -9.8\ m/s^2$ $= -32\ ft/s^2$

$s(t) = \frac{1}{2}(-32)t^2 - 22t + 220$

$s(t) = -16t^2 - 22t + 220$

$v(t) = s'(t) = -32t - 22$

$v(3) = -32(3) - 22 = -96 - 22 = \boxed{-118\ ft/s}$

$108 = -16t^2 - 22t + 220$

$16t^2 + 22t - 112 = 0$

$2(8t^2 + 11t - 56) = 0$

$8t^2 + 11t - 56 = 0 \Rightarrow t \approx 2.046\ s$

$v(2.046) = -32(2.046) - 22 = \boxed{-87.5\ ft/s}$

sphere volume: $V = \frac{4}{3}\pi r^3$

find the rate of change of volume w.r.t. radius when $r = 2$ cm.

$V(r) = \frac{4}{3}\pi r^3$

$V'(r) = 4\pi r^2$ = surface area of a sphere

$V'(2) = 4\pi(2)^2 = \boxed{16\pi \text{ cm}^2}$

HW #3 (due test day, Fri. 8/29)

- **Test #1 Practice Problems**
- **Ch 1 review pp. 88-89 even problems only**
- (*recommended* - Old Test #1 on web; *solutions can be found in course notes from last term*)
- **2.1 #1-23odd -** Find the derivative by the limit process
  #29-32 all - find the equation of the tangent line
  #61-69 odd - Use the alternate form to find the derivative
  #71-79 odd - Describe the x-values where the function is differentiable (given a graph)
- **2.2 #3-51 odd** - Find the derivative using the basic derivative rules
  #91-94 all; 101, 102 - use the derivative to solve rate of change word problems

Test #1 - Fri 8/29

Recommended: Work through intuitive exercises on Khan Academy:

- Slope of secant lines
- Tangent lope is limiting value of secant slope
- Derivative intuition
- Visualizing derivatives
- Graphs of functions and their derivatives
- The formal and alternate form of the derivative
- Derivatives 1
- Recognizing slopes of curves
- Power rule
- Special derivatives