1. $$\frac{-2|x-4|}{4-x} = \begin{cases} \dfrac{-2(x-4)}{4-x} = 2, & x-4>0 \quad x>4 \\ \dfrac{-2[-(x-4)]}{4-x} = -2 & x-4<0 \quad x<4 \end{cases}$$

5. $$\lim_{x\to 0} \frac{6\sin 2x}{3x} = \lim_{x\to 0} \underbrace{\frac{\sin 2x}{2x}}_{1} \cdot \underbrace{\frac{6}{3}\cdot\frac{2}{1}}_{4} = \boxed{4}$$

6. $\varepsilon > 0$; $\delta > 0$

$|-3| = 3$

$|-3(x+2)| = 3|x+2|$

$L = 1$; $\delta = \varepsilon/3$

$x-4$:

rem : $x=4$

non : $x=-8$ & $x-5$

cts on: $(-\infty,-8)\cup(-8,4)\cup(4,5)\cup(5,\infty)$

$x+4$:

no rem

non: $x=-8,-4,5$

cts: $(-\infty,-\infty)\cup(-8,-4)\cup(-4,5)\cup(5,\infty)$

A. $f(x)=\dfrac{(x-4)(x+9)|x+1||x-7|}{(x-4)(x+9)(x+1)(x-7)(x-1)(x+4)}$

$$f(x)=\begin{cases}\dfrac{x+9}{(x+9)(x+4)}, & x<-1\\ \dfrac{x-4}{(x-4)(x-1)}, & -1<x<7\\ 5, & x>7\end{cases}$$

B. $|x+3| \geq 5$

$x+3 \geq 5$ or $x+3 \leq -5$

$x \geq 2$ $\quad$ $x \leq -8$

$-8+10=(-8)^2+b(-8)+c \quad ; \quad 2+10=2^2+b(2)+c$

$b=7$

$c=-6$

2.3 Product & Quotient Rules

$$[fg]' = \frac{d}{dx}[f(x)g(x)] = f'(x)g(x)+f(x)g'(x)$$

$$\boxed{(fg)' = f'g + fg'}$$

$$\left[\frac{f}{g}\right]' = \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$$

"low dee high less high dee low, draw the line and square below"

$$\boxed{\left(\frac{f}{g}\right)' = \frac{f'g-fg'}{g^2}}$$

2.3

6. $g(x) = (\sqrt{x})(\sin x) = (x^{1/2})(\sin x)$

$$g'(x) = (x^{1/2})'(\sin x) + (x^{1/2})(\sin x)'$$

$$= \boxed{\left(\tfrac{1}{2}x^{-1/2}\right)(\sin x) + (x^{1/2})(\cos x)}$$

$$= \frac{\sin x}{2\sqrt{x}} + \sqrt{x}\cos x$$

12. $f(t) = \dfrac{\cos t}{t^3}$

$$f'(t) = \frac{(t^3)(\cos t)' - (\cos t)(t^3)'}{[t^3]^2}$$

$$= \boxed{\frac{t^3(-\sin t) - (\cos t)(3t^2)}{t^6}}$$

$$= \frac{-t\sin t - 3\cos t}{t^4}$$

26. $f(x) = \dfrac{x^3+3x+2}{x^2-1}$

Note: as a product,

$f(x) = (x^3+3x+2)(x^2-1)^{-1}$

we don't know how to differentiate this yet so we have to use the quotient rule!

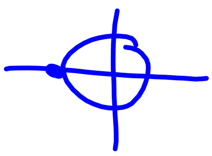
$$f'(x) = \boxed{\dfrac{(x^2-1)(3x^2+3) - (x^3+3x+2)(2x)}{(x^2-1)^2}}$$

Find the slope of the tangent line

$f(x) = 3x - \sin x \quad ; (\pi, 3\pi)$

$f'(x) = 3 - \cos x$

$m = f'(\pi) = 3 - \cos\pi$



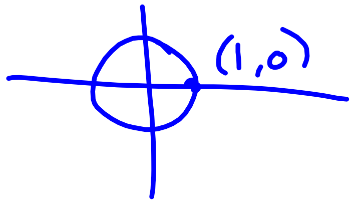
$= 3 - (-1)$

$= \boxed{4}$

Find the equation of the tangent line.

$f(x) = 2x^3 + \sin x - 2x \;;\; (0,0)$

$f'(x) = 6x^2 + \cos x - 2$



$m = f'(0) = 0 + 1 - 2 = -1$

$y - y_1 = m(x - x_1)$

$y = mx$

$\boxed{y = -x}$

2.1

32. $f(x) = \dfrac{1}{x+1} \;;\; (0,1) = (x_1, y_1)$

find the equation of the tangent line

$f'(x) = \dfrac{(x+1)(1)' - (1)(x+1)'}{(x+1)^2}$

$= \dfrac{-1}{(x+1)^2}$

$m = \dfrac{-1}{(0+1)^2} = -1$

$\boxed{y = -x + 1}$

Find f'(x)

2.2

43. $f(x) = \dfrac{x^3 - 3x^2 + 4}{x^2}$

$= \dfrac{x^3}{x^2} - \dfrac{3x^2}{x^2} + \dfrac{4}{x^2}$

$f(x) = x - 3 + 4x^{-2}$

$f'(x) = \boxed{1 - 8x^{-3}}$

$f(x) = \tan x = \dfrac{\sin x}{\cos x}$

$f'(x) = \dfrac{(\cos x)(\sin x)' - (\sin x)(\cos x)'}{(\cos x)^2}$

$= \dfrac{\cos^2 x + \sin^2 x}{\cos^2 x}$

$= \dfrac{1}{\cos^2 x}$

$= \boxed{\sec^2 x}$

Power Rule:
$$\frac{d}{dx}[x^n] = nx^{n-1}$$

Product Rule:
$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

Constant Multiple Rule:
$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$$

Quotient Rule:
$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$$

Sum & Difference:
$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$$

Chain Rule:
$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

Trig Functions:

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\tan x] = \sec^2 x \qquad \frac{d}{dx}[\sec x] = \sec x \tan x$$
$$\frac{d}{dx}[\cos x] = -\sin x \qquad \frac{d}{dx}[\cot x] = -\csc^2 x \qquad \frac{d}{dx}[\csc x] = -\csc x \cot x$$

2.4 - The Chain Rule

$$[f(g(x))]' = f'(g(x)) \cdot g'(x) \cdot x'$$

$$[h(g(f(x)))]' = h'(g(f(x))) \cdot g'(f(x)) \cdot f'(x) \cdot x'$$

$$f(x) = \sin(x^5 - 3x^2)$$

$$f'(x) = \cos(x^5 - 3x^2) \cdot [x^5 - 3x^2]'$$

$$\boxed{= [\cos(x^5 - 3x^2)] \cdot (5x^4 - 6x)}$$

$$f(x) = \cos[5\sin(7x)]$$

$$f'(x) = \boxed{-\sin[5\sin(7x)] \cdot [5\cos(7x)] \cdot 7}$$

$$f(x) = [5x] \cdot [\sin(x^2)]$$

$$f'(x) = [5x]'[\sin(x^2)] + [5x][\sin(x^2)]' =$$

$$= \boxed{5\sin(x^2) + 5x\left(\cos(x^2)\right)(2x)}$$

Homework for Test 2 on Derivatives

Homework #4 (due Fri, 09/05)

- 2.2 #3-51 odd — Find derivative using basic rules
- 2.2 #91-94 all; 101,102 — Use derivative to solve rate of change word problems
- 2.3 #1-53 odd, 63-69 odd, 75-81 all, 83-91 odd, 109-115 all — Product and quotient rules
- 2.4 #7-33 odd — Chain rule

Homework #5

- 2.4 #47-81 odd — Chain rule
- 5.1 #45-61, 71 — Logarithmic functions
- 5.4 #39-57 — Exponential functions
- 5.5 #41-55 — Log and exp functions with other bases
- 5.8 #41-59 — Inverse trig functions

Test 2 - on the syllabus for Friday, 09/26, but we will be ready sooner!

Quiz Friday on basic derivatives