What happens if . . .

$$x^2y + y^2x = -2$$

how to find y'?

2.5 Implicit Differentiation

★ $y = f(x)$

y is a function of x

$$\frac{d}{dx}[x] = 1 \quad ; \quad \frac{d}{dx}[y] = y'$$

6. $x^2y + y^2x = -2$

$$\frac{d}{dx}\left[x^2y + y^2x\right] = \frac{d}{dx}\left[-2\right]$$

$$(x^2)'y + x^2(y)' + (y^2)'x + y^2(x)' = 0$$

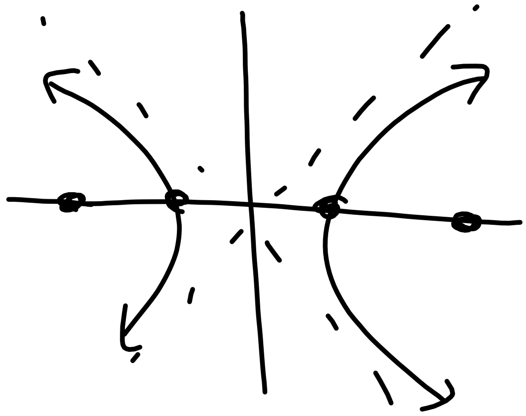
$$2xy + x^2y' + 2y\cdot y'x + y^2 = 0$$

$$x^2y' + 2xyy' = -y^2 - 2xy$$

$$y'(x^2 + 2xy) = -y^2 - 2xy$$

$$\boxed{y' = \frac{-y^2 - 2xy}{x^2 + 2xy}}$$

2. $x^2 - y^2 = 16$



$$x^2 - 16 = y^2$$

$$\pm\sqrt{x^2 - 16} = y$$

$$\frac{d}{dx}\left[x^2 - y^2\right] = \frac{d}{dx}\left[16\right]$$

$$2x - 2yy' = 0$$

$$-2yy' = -2x$$

$$y' = \frac{-2x}{-2y} = \boxed{\frac{x}{y}}$$

8. $\sqrt{xy} = x - 2y$

$$(xy)^{1/2} = x - 2y$$

$$\frac{d}{dx}\left[(xy)^{1/2}\right] = \frac{d}{dx}\left[x - 2y\right]$$

$$\frac{1}{2}(xy)^{-1/2} \cdot \left[1 \cdot y + xy'\right] = 1 - 2y'$$

$$\frac{y}{2}(xy)^{-1/2} + \frac{xy'}{2}(xy)^{-1/2} = 1 - 2y'$$

$$\frac{xy'}{2}(xy)^{-1/2} + 2y' = 1 - \frac{y}{2}(xy)^{-1/2}$$

$$y'\left[\frac{x}{2}(xy)^{-1/2} + 2\right] = 1 - \frac{y}{2}(xy)^{-1/2}$$

$$y' = \boxed{\frac{1 - \frac{y}{2}(xy)^{-1/2}}{\frac{x}{2}(xy)^{-1/2} + 2}} = \frac{1 - \frac{y}{2\sqrt{xy}}}{\frac{x}{2\sqrt{xy}} + 2}$$

$$= \frac{\frac{2\sqrt{xy} - y}{2\sqrt{xy}}}{\frac{x + 4\sqrt{xy}}{2\sqrt{xy}}} = \frac{2\sqrt{xy} - y}{\cancel{2\sqrt{xy}}} \cdot \frac{\cancel{2\sqrt{xy}}}{x + 4\sqrt{xy}}$$

$$= \boxed{\frac{2\sqrt{xy} - y}{x + 4\sqrt{xy}}} \cdot \frac{x - 4\sqrt{xy}}{x - 4\sqrt{xy}} = \frac{2x\sqrt{xy} - 8xy - xy + 4y\sqrt{xy}}{x^2 - 16xy}$$

10. $2\sin x \cos y = 1$

$$\frac{d}{dx}\left[2\sin x\cos y\right] = \frac{d}{dx}\left[1\right]$$

$$2\cos x\cos y + 2\sin x\left(-\sin y \cdot y'\right) = 0$$

$$2\cos x\cos y = 2y'\sin x\sin y$$

$$\frac{2\cos x\cos y}{2\sin x\sin y} = y'$$

$$\cot x\cot y = y'$$

12. $(\sin\pi x + \cos\pi y)^2 = 2$

$$\frac{d}{dx}\left[(\sin\pi x + \cos\pi y)^2\right] = \frac{d}{dx}[2]$$

$$2(\sin\pi x + \cos\pi y)\cdot(\pi\cos\pi x - \pi y'\sin\pi y) = 0$$

$$2\pi\cos\pi x(\sin\pi x + \cos\pi y) - 2\pi y'\sin\pi y(\sin\pi x + \cos\pi y) = 0$$

$$\frac{2\pi\cos\pi x(\sin\pi x + \cos\pi y)}{2\pi\sin\pi y(\sin\pi x + \cos\pi y)} = y'$$

$$\boxed{\frac{\cos\pi x}{\sin\pi y} = y'}$$

16. $x = \sec\frac{1}{y}$

$$(y^{-1})' = -y^{-2}\cdot y'$$

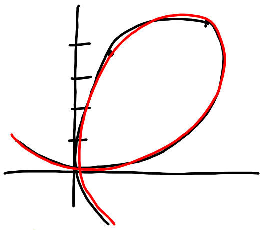
$$\frac{d}{dx}[x] = \frac{d}{dx}\left[\sec\frac{1}{y}\right]$$

$$1 = \sec\frac{1}{y}\tan\frac{1}{y}\cdot\left(\frac{-y'}{y^2}\right)$$

$$\frac{-y^2}{\sec\frac{1}{y}\tan\frac{1}{y}} = y'$$

$$\boxed{-y^2\cos\frac{1}{y}\cot\frac{1}{y} = y'}$$

32. Folium of Descartes



$x^3+y^3-6xy=0$

find the slope of the tangent line @ $\left(\frac{4}{3}, \frac{8}{3}\right)$

$$\frac{d}{dx}\left[x^3+y^3-6xy\right]=\frac{d}{dx}[0]$$

$$3x^2+3y^2y'-6\left[xy'+1\cdot y\right]=0$$

$$3x^2+3y^2y'-6xy'-6y=0$$

$$3y^2y'-6xy'=6y-3x^2$$

$$y'=\frac{6y-3x^2}{3y^2-6x}=\frac{2y-x^2}{y^2-2x}$$

$$y'\Big|_{(4/3,\,8/3)}=\frac{2\left(\frac{8}{3}\right)-\left(\frac{4}{3}\right)^2}{\left(\frac{8}{3}\right)^2-2\left(\frac{4}{3}\right)}=\frac{\frac{16}{3}-\frac{16}{9}}{\frac{64}{9}-\frac{8}{3}}=\frac{\frac{48-16}{9}}{\frac{64-24}{9}}$$

$$=\frac{32}{40}=\boxed{\frac{4}{5}}$$

Homework since Test #2 (Material for Test #3)

2.5 # 1-39 odd; 43, 47 - Implicit Differentiation
2.6 # 15-23 odd - Related Rates
2.6 # 25, 27, 35 - Related Rates (more challenging problems)

3.1 # 17-31 odd - Absolute Extrema on an Interval
3.2 # 7-19 odd - Rolle's Theorem
3.2 # 31-37 odd - Mean Value Theorem
3.3 # 11-31 odd - Increasing, Decreasing, and Relative Extrema
3.4 #11-25 odd - Inflection Points and Concavity