1. $f(x) = \sin(\log_2 x + x^6)$

$f'(x) = \cos(\log_2 x + x^6) \cdot \left(\frac{1}{x \ln 2} + 6x^5\right)$

2. $f(x) = \arccos(\ln x) \tan x$

$f'(x) = \arccos(\ln x) \cdot \sec^2 x + \frac{-1}{\sqrt{1-(\ln x)^2}} \cdot \frac{1}{x} \cdot \tan x$

3. $f(x) = \frac{\csc x}{3^{\arctan x}}$

$f'(x) = \frac{3^{\arctan x} \cdot (-\csc x \cot x) - \csc x \cdot 3^{\arctan x} \cdot \ln 3 \cdot \frac{1}{1+x^2}}{(3^{\arctan x})^2}$

4. $f(x) = \frac{\operatorname{arccot} x}{e^x + \cos x}$

$f'(x) = \frac{(e^x + \cos x) \cdot \frac{-1}{1+x^2} - (\operatorname{arccot} x)(e^x - \sin x)}{(e^x + \cos x)^2}$

5. $f(x) = \operatorname{arcsec}(3x^5) - \cot x$

$f'(x) = \frac{1}{|3x^5|\sqrt{(3x^5)^2 - 1}} \cdot 15x^4 - (-\csc^2 x)$

6. $f(x) = \sec^2(\arcsin x) = [\sec(\arcsin x)]^2$

$f'(x) = 2\sec(\arcsin x) \cdot \sec(\arcsin x)\tan(\arcsin x) \cdot \frac{1}{\sqrt{1-x^2}}$

7. $f(x) = \ln\left(\sqrt[3]{\frac{2x}{x-3}}\right)$ $= \ln\left(\frac{2x}{x-3}\right)^{1/3} = \frac{1}{3}\ln\left(\frac{2x}{x-3}\right) =$

$= \frac{1}{3}\left[\ln 2x - \ln(x-3)\right]$

$f'(x) = \frac{1}{3}\left(\frac{1}{2x}\cdot 2 - \frac{1}{x-3}\right)$

8. $f(x) = \sqrt[5]{\operatorname{arccsc}(\pi x)}$ $= (\operatorname{arccsc}\pi x)^{1/5}$

$f'(x) = \frac{1}{5}(\operatorname{arccsc}\pi x)^{-4/5} \cdot \frac{-1}{|\pi x|\sqrt{(\pi x)^2-1}} \cdot \pi$

9. $f(x) = \frac{x^5-2x^4+x^2-3x+1}{2x}$ $= \frac{1}{2}x^4 - x^3 + \frac{1}{2}x - \frac{3}{2} + \frac{1}{2}x^{-1}$

$f'(x) = 2x^3 - 3x^2 + \frac{1}{2} - \frac{1}{2}x^{-2}$

10. $f(x) = 4\sin(3x)\cos(3x)$ $= 2\sin 6x$

$f'(x) = 2\cos 6x \cdot 6$

$= 12\cos 6x$

$f(x) = \ln\sqrt{1-\cos^2\left(\arcsin\left(\frac{1}{\csc x}\right)\right)}$

$= \ln\left[1-\cos^2(\underline{\arcsin(\sin x)})\right]^{1/2}$

$= \ln\left[1-\cos^2 x\right]^{1/2}$

$= \ln\left[\sin^2 x\right]^{1/2} = \ln\left[(\sin x)^2\right]^{1/2} = \ln(\sin x)$

$f'(x) = \frac{1}{\sin x}\cdot\cos x = \boxed{\cot x}$

$$f(x) = \ln\sqrt{1-\cos^2\left(\arcsin\left(\frac{1}{\csc x}\right)\right)}$$

$$f'(x) = \frac{1}{\sqrt{1-\cos^2(\arcsin\frac{1}{\csc x})}} \cdot \frac{1}{2}\left(1-\cos^2\left(\arcsin\frac{1}{\csc x}\right)\right)^{-1/2} \cdot$$

$$\left(-2\cos\left(\cancel{\arcsin\frac{1}{\csc x}}\right)\right)\left(-\sin\left(\cancel{\arcsin\frac{1}{\csc x}}\right)\right) \cdot$$

$$\frac{1}{\sqrt{1-\left(\frac{1}{\csc x}\right)^2}} \cdot \left(-(\csc x)^{-2}\right)\left(-\csc x \cot x\right)$$

$$= \frac{1}{\sqrt{\sim\sim}} \cdot \frac{1}{2\sqrt{\sim\sim}} \cdot 2\cos x \sin x \cdot \frac{1}{\sqrt{1-\sin^2 x}} \cdot \frac{\cot x}{\csc x}$$

$$= \frac{1}{2\sin^2 x} \cdot 2\cos x \sin x \cdot \frac{1}{\sqrt{\cos^2 x}} \cdot \frac{\cot x}{\csc x}$$

$$= \frac{1}{\cancel{2\sin^2 x}} \cdot \cancel{2\cos x \sin x} \cdot \frac{1}{\cancel{\cos x}} \boxed{\cdot \cot x} \cdot \sin x$$

40. Find $y''$ in terms of $x$ & $y$.

$y^2 = 4x$

$\frac{d}{dx}[y^2] = \frac{d}{dx}[4x]$

$2y \cdot y' = 4$

$y' = \frac{4}{2y} = \frac{2}{y}$

$\boxed{y' = 2y^{-1}}$

$\frac{d}{dx}[y'] = \frac{d}{dx}[2y^{-1}]$

$y'' = -2y^{-2}\boxed{y'}$

$= -2y^{-2}(2y^{-1})$

$= -4y^{-3}$

$\boxed{y'' = -\frac{4}{y^3}}$

# 2.6 Related Rates

18. $V = \frac{4}{3}\pi r^3$

$\frac{dr}{dt} = 2 \text{ in/min}$

$\frac{dV}{dt} = ?$ when $r = 6$ in

$$\frac{d}{dt}[V] = \frac{d}{dt}\left[\frac{4}{3}\pi r^3\right]$$

$$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$$

$$= 4\pi (6 \text{ in})^2 \cdot \left(\frac{2 \text{ in}}{\text{min}}\right) = \boxed{288\pi \ \frac{\text{in}^3}{\text{min}}}$$

22. $V = \frac{1}{3}\pi r^2 h$

$\frac{dr}{dt} = 2 \text{ in/min}$

$h = 3r$

$\frac{dV}{dt} = ?$ when $r = 6$in

$$\frac{d}{dt}[V] = \frac{d}{dt}\left[\frac{1}{3}\pi r^2 h\right]$$

$$\frac{dV}{dt} = \frac{1}{3}\pi r^2 \frac{dh}{dt} + \frac{1}{3}\pi h \cdot 2r \cdot \frac{dr}{dt}$$

don't want this!

rewrite $V = \frac{1}{3}\pi r^2 (3r)$

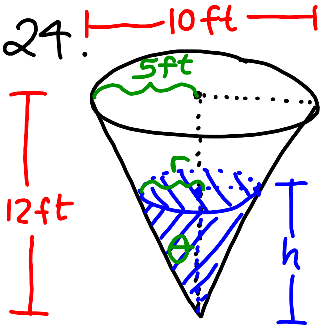
$$V = \pi r^3$$

$$\frac{d}{dt}[V] = \frac{d}{dt}[\pi r^3]$$

$$\frac{dV}{dt} = 3\pi r^2 \cdot \frac{dr}{dt} = 3\pi (6 \text{ in})^2 \cdot (2 \text{ in/min})$$

$$= \boxed{216\pi \text{ in}^3/\text{min}}$$

24.




$$\frac{dV}{dt} = 10\ \text{ft}^3/\text{min}$$

$$\frac{dh}{dt} = ? \text{ when } h = 8 \text{ ft}$$

$$\frac{5}{12} = \frac{r}{h} \Rightarrow r = \frac{5h}{12}$$

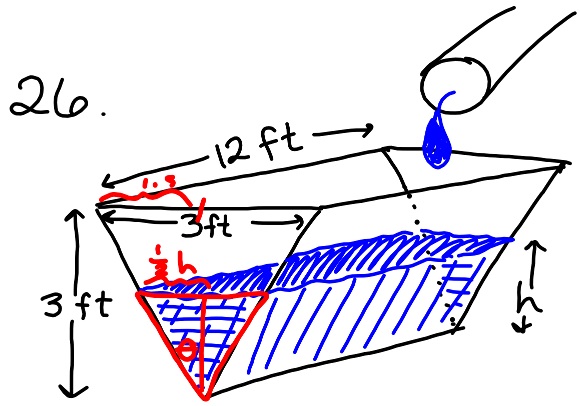
$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{5h}{12}\right)^2 \cdot h$$

$$V = \frac{1}{3}\cdot\frac{25\pi}{144} h^3$$

$$\frac{dV}{dt} = \frac{25\pi}{144} h^2 \cdot \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{\frac{dV}{dt}}{\frac{25\pi}{144} h^2} = \frac{\cancel{10}^{2}\cdot \overset{3\cdot\cancel{4}\cdot 3\cdot\cancel{4}}{144}}{{}_{5}\cancel{25}\pi\underset{2\cdot\cancel{4}\cdot 2\cdot\cancel{4}}{(8)^2}} = \boxed{\frac{9}{10\pi}\ \text{ft}/\text{min}}$$

26.




(a) $\frac{dV}{dt} = 2\ \text{ft}^3/\text{min}$

$$\frac{dh}{dt} = ? \text{ when } h = 1 \text{ ft}$$

V = area of ▽ × 12

$$V = \frac{1}{2}h^2 \cdot 12$$

$$V = 6h^2$$

$$\frac{dV}{dt} = 12h \cdot \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{12h}{\frac{dV}{dt}} = \frac{12\cdot 1}{2} = \boxed{6\ \text{ft}/\text{min}}$$

Homework since Test #2 (Material for Test #3)

2.5 # 1-39 odd; 43, 47 - Implicit Differentiation ✓ → due Wed
2.6 # 15-23 odd - Related Rates ✓
2.6 # 25, 27, 35 - Related Rates (more challenging problems)

3.1 # 17-31 odd - Absolute Extrema on an Interval
3.2 # 7-19 odd - Rolle's Theorem
3.2 # 31-37 odd - Mean Value Theorem
3.3 # 11-31 odd - Increasing, Decreasing, and Relative Extrema
3.4 #11-25 odd - Inflection Points and Concavity