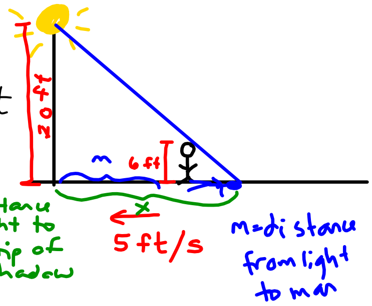
36. A man 6 ft tall walks toward a light that is 20 ft above the ground at a rate of 5 ft/s. When he is 10 ft from the base of the light,

(a) at what rate is the tip of his shadow moving? $\frac{dx}{dt} = ?$ x = distance from light to tip of shadow

(b) at what rate is the length of his shadow changing?



(a) $\frac{20}{x} = \frac{6}{s}$

$20s = 6x$

$10s = 3x$

$x = \frac{10}{3}s$

$\frac{dx}{dt} = \frac{10}{3} \cdot \frac{ds}{dt}$

$\frac{20}{x} = \frac{6}{x-m}$

$20(x-m) = 6x$

$20x - 20m = 6x$

$14x = 20m$

$x = \frac{10}{7}m$

$\frac{dx}{dt} = \frac{10}{7} \cdot \frac{dm}{dt}$

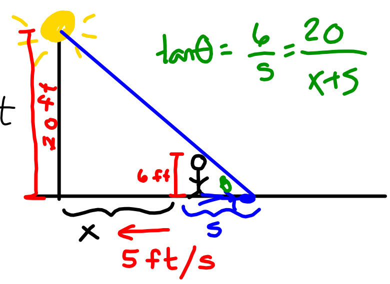
$= \frac{10}{7}(-5\text{ ft/s})$

$= \boxed{-\frac{50}{7}\text{ ft/s}}$

36. A man 6 ft tall walks toward a light that is 20 ft above the ground at a rate of 5 ft/s. When he is 10 ft from the base of the light,

~~(a) at what rate is the tip of his shadow moving?~~

(b) at what rate is the length of his shadow changing?



$\tan\theta = \frac{6}{s} = \frac{20}{x+s}$

Let s = length of shadow

want to know $\frac{ds}{dt}$ when $x = 10$

x = distance from man to light

$\frac{dx}{dt} = -5$ ft/s

$\frac{20}{x+s} = \frac{6}{s}$

$20s = 6(x+s)$

$20s = 6x + 6s$

$14s = 6x$

$s = \frac{3x}{7}$

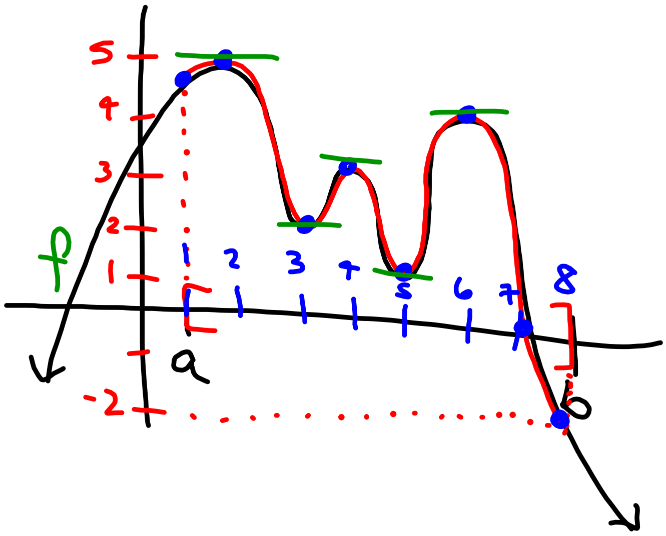
$\frac{ds}{dt} = \frac{3}{7} \cdot \frac{dx}{dt}$

$\frac{ds}{dt} = \frac{3}{7}(-5) =$

$= \boxed{-\frac{15}{7}\text{ ft/s}}$

3.1 Extrema on an Interval

- maxima & minima
- relative & absolute



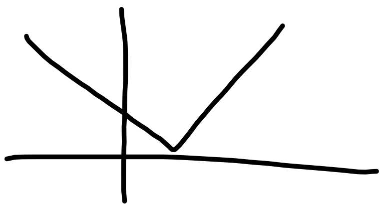
relative minima:
(3,2), (5,1)

relative maxima:
(2,5), (4,3), (6,4)

absolute maximum:
5 @ (2,5)

absolute minimum:
-2 @ (8,-2)

f(x) has a relative maximum or minimum when f'(x) = 0. or



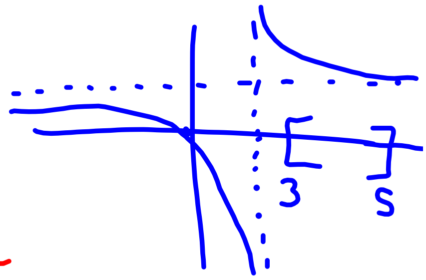
f'(x) is undefined.

We call such x-values

Critical #'s of f.

3.1

28. $h(t) = \dfrac{t}{t-2}$, $[3,5]$



Find the absolute max & min on the closed interval.

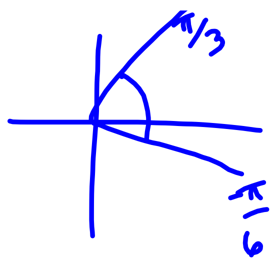
$$h'(t) = \frac{(t-2)(1) - t(1)}{(t-2)^2} = \frac{-2}{(t-2)^2}$$

critical #'s : 2 (h is undefined here)

$h(3) = \frac{3}{3-2} = 3 \leftarrow$ absolute max

$h(5) = \frac{5}{5-2} = \frac{5}{3} \leftarrow$ absolute min

30. $g(x) = \sec x$, $\left[-\frac{\pi}{6}, \frac{\pi}{3}\right]$



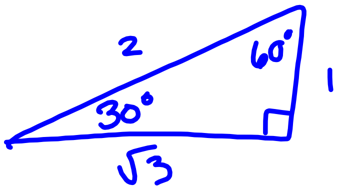
Find the absolute max & min on the closed interval.

$$g'(x) = \sec x \tan x = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} = \frac{\sin x}{(\cos x)^2}$$

$\sin x = 0$ when $x = 0$

$\cos x \neq 0$ in this interval

critical # : 0



$\sec\left(-\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}$

$1 < 3 < 4$

$\frac{1}{2} < \frac{\sqrt{3}}{2} < \frac{2}{2}$

$\sec 0 = 1 \leftarrow$ abs min

$2 > \frac{2}{\sqrt{3}} > 1$

$\sec \frac{\pi}{3} = 2 \leftarrow$ abs max

22. $f(x) = x^3 - 12x \quad , \quad [0, 4]$

Find the absolute max & min on the closed interval.

$f'(x) = 3x^2 - 12 = 3(x^2-4) = 3(x-2)(x+2)$

critical #'s in interval: 2

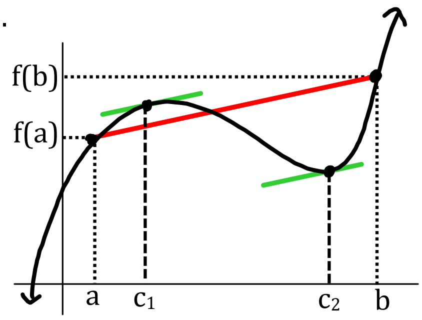
$f(0) = 0^3 - 12(0) = 0$

$f(2) = 2^3 - 12(2) = 8 - 24 = -16 \leftarrow$ abs min

$f(4) = 4^3 - 12(4) = 4(16-12) = 16 \leftarrow$ abs max

3.2 Rolle's Theorem & The Mean Value Theorem

The Mean Value Thereom (MVT) states: If f is continuous on [a,b] and differentiable on (a,b), then there exists at least one c in (a,b) such that the slope of the tangent line at c is equal to the slope of the secant line through (a, f(a)) and (b, f(b)).

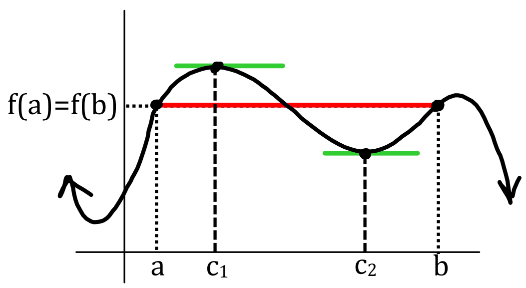


slope of secant line:

$$\frac{f(b)-f(a)}{b-a} = f'(c)$$

slope of tangent line

Rolle's Theorem is a special case of the MVT where f(a)=f(b), (and hence involving horizontal secant/tangent lines)



Note that neither the Mean Value Theorem nor Rolle's Theorem apply to the following functions on the given intervals:

$$f(x) = \frac{x+5}{x-2}, \qquad [1,3]$$

f is not continuous on [1,3].

$$g(x) = |x-2|, \qquad [1,3]$$

g is continuous on [1,3], but not differentiable on (1,3).

Can Rolle's Theorem be applied?
If so, find all guaranteed values of c in (a,b).

8. $f(x) = x^2 - 5x + 4, \quad [1,4]$

$\left.\begin{array}{l} f \text{ cts on } [1,4]? \\ f \text{ diff on } (1,4)? \end{array}\right\}$ yes

$\left.\begin{array}{l} f(1) = 1^2 - 5(1) + 4 = 0 \\ f(4) = 4^2 - 5(4) + 4 = 0 \end{array}\right\}$ $f(1) = f(4)$ ✓

Rolle's Theorem can be applied

$f'(x) = 2x - 5$

$f'(c) = 0$

$2c - 5 = 0$

$2c = 5$

$\boxed{c = 5/2}$

Can the Mean Value Theorem be applied?
If so, find all guaranteed values of c in (a,b).

34. $f(x) = \frac{x+1}{x}$, $[\frac{1}{2}, 2]$

Steps to solve MVT problems:

1. Is f continuous on [a,b]? yes
2. Is f differentiable on (a,b)? yes } MVT applies
3. Find (f(b)-f(a))/(b-a) slope of secant line
4. Find f'(x) slope of tangent line
5. Set #3&4 equal, solve for x
6. Solution is the values of x from #5 that lie in (a,b)

$$\frac{f(b)-f(a)}{b-a} = \frac{\frac{2+1}{2} - \frac{\frac{1}{2}+1}{\frac{1}{2}}}{2-\frac{1}{2}} = \frac{\frac{3}{2} - \frac{3}{\cancel{2}}\cdot\frac{\cancel{2}}{1}}{\frac{3}{2}} = \frac{-\frac{3}{2}}{\frac{3}{2}} = -1$$

$$f'(x) = \frac{x(1)-(x+1)(1)}{x^2} = \frac{-1}{x^2}$$

$\frac{-1}{x^2} = -1$ -1 is not in $[\frac{1}{2}, 2]$

$x^2 = 1$ so, the only answer is $\boxed{1}$

$x = \pm 1$

38. $f(x) = 2\sin x + \sin 2x$, $[0, \pi]$

Does MVT apply?

f cts on $[0,\pi]$? yes } MVT applies
f diff on $(0,\pi)$? yes

$$\frac{f(b)-f(a)}{b-a} = \frac{(2\sin\pi + \sin 2\pi) - (2\sin 0 + \sin 2(0))}{\pi - 0}$$

$$= \frac{0}{\pi} = 0$$

$f'(x) =$

Homework since Test #2 (Material for Test #3)

2.5 # 1-39 odd; 43, 47 - Implicit Differentiation ✓ → due Wed
2.6 # 15-23 odd - Related Rates ✓
2.6 # 25, 27, 35 - Related Rates (more challenging problems) ✓ next Fri

3.1 # 17-31 odd - Absolute Extrema on an Interval ✓
3.2 # 7-19 odd - Rolle's Theorem ✓
3.2 # 31-37 odd - Mean Value Theorem ✓
3.3 # 11-31 odd - Increasing, Decreasing, and Relative Extrema
3.4 #11-25 odd - Inflection Points and Concavity