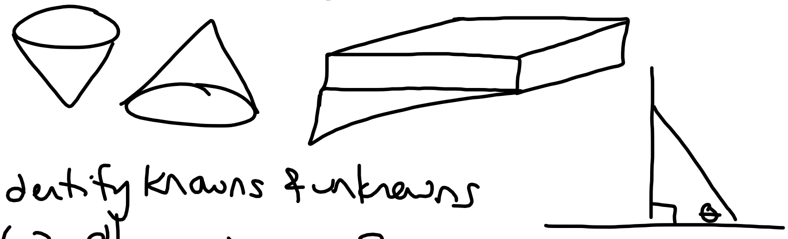
1. draw a picture
(identify geometry of situation)

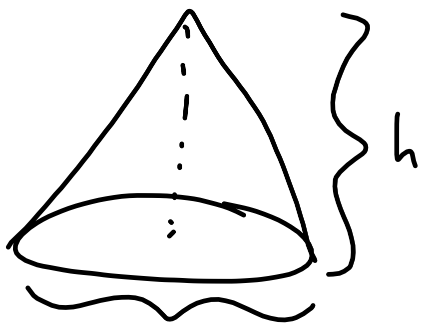


2. identify knowns & unknowns
$\frac{dV}{dt}$? $\frac{dh}{dt}$, when $r=2$
$r=0$ when $h=0$

3. Identify relationship between variables
$V=\frac{1}{3}\pi r^2 h$; $V=(\frac{1}{2}bh\cdot l)+(bhl)$; $A=\frac{1}{2}bh$

4. compare relationship in #3 to derivative variables in #2
& rewrite any variables in terms of other variables as necessary (* don't plug in constants yet)

5. do implicit differentiation & solve for unknown



$d=2r$

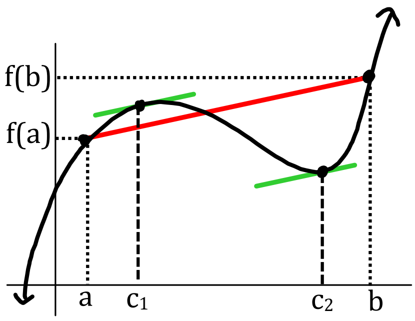
$2r=3h$

$r=\frac{3h}{2}$

$$V=\frac{1}{3}\pi r^2 h=\frac{1}{3}\pi\left(\frac{3h}{2}\right)^2 h=\frac{1}{3}\cdot\frac{9\pi}{4}h^3$$

3.2 Rolle's Theorem & The Mean Value Theorem

The Mean Value Thereom (MVT) states: If f is continuous on [a,b] and differentiable on (a,b), then there exists at least one c in (a,b) such that the slope of the tangent line at c is equal to the slope of the secant line through (a, f(a)) and (b, f(b)).

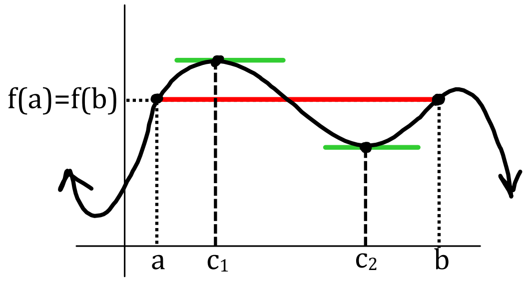


slope of secant line:

$$\frac{f(b)-f(a)}{b-a} = f'(c)$$

slope of tangent line

Rolle's Theorem is a special case of the MVT where f(a)=f(b), (and hence involving horizontal secant/tangent lines)



38. $f(x)=2\sin x+\sin 2x$, $[0,\pi]$

Does MVT apply?

f cts on $[0,\pi]$? yes
f diff on $(0,\pi)$? yes } MVT applies

$$\frac{f(b)-f(a)}{b-a} = \frac{(2\sin\pi+\sin 2\pi)-(2\sin 0+\sin 2(0))}{\pi-0}$$

$$=\frac{0}{\pi}=0$$

$f'(x)=2\cos x+2\cos 2x$

$2\cos x+2\cos 2x=0$

$2\cos x+2(2\cos^2 x-1)=0$

$4\cos^2 x+2\cos x-2=0$

$2\cos^2 x+\cos x-1=0$

$(2\cos x-1)(\cos x+1)=0$

$\cos x=\frac{1}{2}$ $\cos x=-1$

$\boxed{x=\frac{\pi}{3}}$ $\cancel{x=\pi}$ → not in open interval $(0,\pi)$

32. $f(x) = x(x^2 - x - 2)$ $[-1, 1]$

does MVT apply?

yes, polynomials are cts. & diff. everywhere

$$\frac{f(1)-f(-1)}{1-(-1)} = \frac{-2-(0)}{2} = -1$$

$f(x) = x^3 - x^2 - 2x$

$f'(x) = 3x^2 - 2x - 2$

$3x^2 - 2x - 2 = -1$

$3x^2 - 2x - 1 = 0$

$(3x+1)(x-1) = 0$

$\boxed{x = -1/3}$ $\cancel{x = 1}$

3.3-3.4 Increasing, Decreasing, Concavity, and the 1st and 2nd Derivative Tests

What do f' and f'' tell us about f?

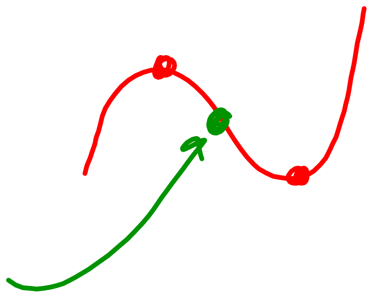
Recall that f' is the rate of change or slope of f,
f'' is the slope or rate of change of f'.

f'	f
+	↗ increasing
−	↘ decreasing

f''	f'	f
+	↗ increasing	concave up
−	↘ decreasing	concave down

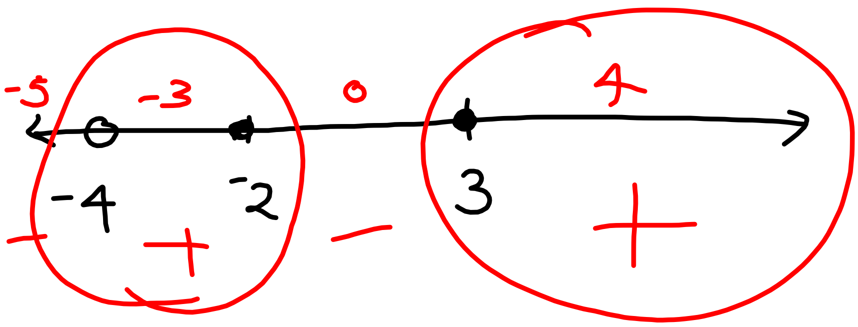
$f'(x)=0$ when f has a relative maximum or minimum.
These x-values (and those where f'(x) is undefined) are called <u>critical numbers</u>.

$f''(x)=0$ when f changes concavity.
The points where concavity changes are called <u>inflection points</u>.



To solve problems involving concavity, increasing/decreasing, etc., we should recall how to solve polynomial inequalities.

$$\frac{(x+2)(x-3)}{x+4} \geq 0 \qquad \boxed{(-4,-2]\cup[3,\infty)}$$



- Find all critical numbers and state the open intervals on which f is increasing and/or decreasing.
- Find all inflection points and state the open intervals on which f is concave up and/or concave down.
- Use these results to determine all relative and absolute extrema.

3.3

16. $f(x) = x^3 - 6x^2 + 15$

$f'(x) = 3x^2 - 12x$

$3x(x-4) = 0$

critical #'s : 0 & 4

$f'(-1)$ $f'(1)$ $f'(5)$

$+$ 0 $-$ 4 $+$

f is increasing on $(-\infty, 0) \cup (4, \infty)$

f is decreasing on $(0, 4)$

$f''(0)$ $f''(3)$

$-$ 2 $+$

$f''(x) = 6x - 12$

$6(x-2) = 0$

inflection points : $(2, f(2)) = (2, -1)$

f is concave down on $(-\infty, 2)$ & concave up on $(2, \infty)$

relative max @ $(0, f(0)) = (0, 15)$

relative min @ $(4, f(4)) = (4, -17)$

no absolute extrema!

Homework #6 (submitted Wed. 9/17)
2.5 # 1-39 odd; 43, 47 - Implicit Differentiation

Homework #7 (due Fri. 9/26)
2.6 # 15-23 odd - Related Rates
2.6 # 25, 27, 35 - Related Rates (more challenging problems)
3.1 # 17-31 odd - Absolute Extrema on an Interval

Homework #8 (due Fri, 10/3)
3.2 # 7-19 odd - Rolle's Theorem
3.2 # 31-37 odd - Mean Value Theorem
3.3 # 11-31 odd - Increasing, Decreasing, and Relative Extrema
3.4 #11-25 odd - Inflection Points and Concavity

Quiz #4 - Mon, 9/29
Test #3 - Fri, 10/3