3.4 #20

$$f(x) = \frac{x+1}{\sqrt{x}}$$

domain: $(0,\infty)$

critical #'s: 0 & 1
f is decreasing on $(0,1)$, increasing on $(1,\infty)$
relative minimum @ $(1,2)$ (also absolute minimum)
inflection point @ $(3, 4/\sqrt{3}) = (3, \frac{4\sqrt{3}}{3})$
f is concave up on $(0,3)$ & concave down $(3,\infty)$

$$f'(x) = \frac{\sqrt{x}(1) - (x+1)(\frac{1}{2}x^{-1/2})}{(\sqrt{x})^2} = \frac{x^{1/2} - \frac{1}{2}x^{1/2} - \frac{1}{2}x^{-1/2}}{x^1}$$

$$= \frac{\frac{1}{2}x^{1/2} - \frac{1}{2}x^{-1/2}}{x^1} \cdot \frac{x^{1/2}}{x^{1/2}} = \frac{\frac{1}{2}x - \frac{1}{2}}{x^{3/2}} = \frac{x-1}{2x^{3/2}}$$

$\cancel{f'(-1)}$ $f'(\frac{1}{2})$, $f'(2)$

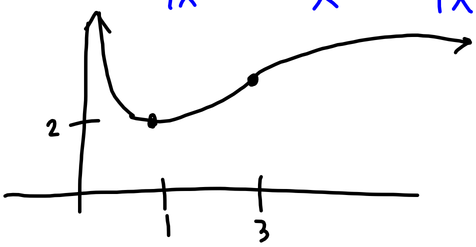
0 $-$ 1 $+$

$$f''(x) = \frac{2x^{3/2}(1) - (x-1)(3x^{1/2})}{(2x^{3/2})^2} =$$

$$f''(x) = \frac{2x^{3/2} - 3x^{3/2} + 3x^{1/2}}{4x^3} = \frac{-x^{3/2} + 3x^{1/2}}{4x^3} \cdot \frac{x^{-1/2}}{x^{-1/2}} = \frac{-x+3}{4x^{5/2}}$$

$f''(1)$, $f''(4)$

0 $+$ 3 $-$



1. Find the average rate of change of the volume of a sphere with respect to radius, as the radius of the sphere changes from 1 cm to 2 cm.

$$V(r) = \frac{4}{3}\pi r^3$$

$$\frac{V(2) - V(1)}{2-1} =$$

$$= \frac{\frac{4}{3}\pi(2)^3 - \frac{4}{3}\pi(1)^3}{1} =$$

$$= \frac{4}{3}\pi(8-1) = \boxed{\frac{28\pi}{3}\text{ cm}^2}$$

The average rate of change of f from a to b is $\frac{f(b)-f(a)}{b-a}$

The instantaneous rate of change of f at c is $f'(c)$

2. Find the instantaneous rate of change of the volume of a sphere with respect to radius when the radius is 2 cm.

$$V(r) = \frac{4}{3}\pi r^3$$

$$\frac{d}{dr}\left[V(r)\right] = \frac{dV}{dr} = V'(r) = 4\pi r^2 \cdot \underbrace{\frac{d}{dr}(r)}_{=1}$$

$$V'(r)\Big|_{r=2cm} = 4\pi(2)^2 = \boxed{16\pi\ cm^2}$$

3. If the radius of a sphere changes at a rate of 3cm per second, find the rate of change of the volume of the sphere with respect to time when the radius is 2 cm.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt} = 4\pi(2cm)^2 \cdot \left(3\ \frac{cm}{s}\right) =$$

$$= \boxed{48\pi\ cm^3/s}$$

Find y' implicitly in terms of x and y.

Review

$$x^2y + 3xy^3 = 5x^3y^2$$

$$2xy + x^2y' + 3y^3 + 9xy^2y' = 15x^2y^2 + 10x^3yy'$$

$$x^2y' + 9xy^2y' - 10x^3yy' = 15x^2y^2 - 2xy - 3y^3$$

$$\boxed{y' = \frac{15x^2y^2 - 2xy - 3y^3}{x^2 + 9xy^2 - 10x^3y}}$$

$$\cos x + \sin y = \tan(xy)$$

$$-\sin x + y'\cos y = \sec^2(xy) \cdot [xy' + y]$$

$$-\sin x + y'\cos y = xy'\sec^2(xy) + y\sec^2(xy)$$

$$y'\cos y - xy'\sec^2(xy) = y\sec^2(xy) + \sin x$$

$$\boxed{y' = \frac{y\sec^2(xy) + \sin x}{\cos y - x\sec^2(xy)}}$$

Homework #6 (submitted Wed. 9/17)
2.5 # 1-39 odd; 43, 47 - Implicit Differentiation

Homework #7 (due Fri. 9/26)
2.6 # 15-23 odd - Related Rates
2.6 # 25, 27, 35 - Related Rates (more challenging problems)
3.1 # 17-31 odd - Absolute Extrema on an Interval

Homework #8 (due Fri. 10/3)
3.2 # 7-19 odd - Rolle's Theorem
3.2 # 31-37 odd - Mean Value Theorem
3.3 # 11-31 odd - Increasing, Decreasing, and Relative Extrema
3.4 #11-25 odd - Inflection Points and Concavity

Quiz #4 - Mon, 9/29
Test #3 - Fri, 10/3