1. x-values for which $f'(x) = 0$ or $f'(x)$ is undefined are called critical #'s.

2. Relative maxima or minima of a function $f(x)$ *can* occur when $f'(x)=0$ or $f'(x)$ is undefined.

3. When determining absolute extrema of a function $f(x)$ on an interval, all critical numbers located in the interval and endpoint values of the interval should be plugged into $f(x)$ in order to compare their respective y-values.

4. What conditions must a function $f(x)$ fulfill in order for the Mean Value Theorem to guarantee a c in (a, b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$?

   f is continuous on $[a,b]$
   & differentiable on (a,b)

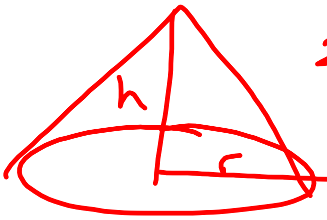
5. Find y' implicitly, and then give y' in terms of x and y.

$$x^2 + y^2 = 36$$

$$2x + 2y \cdot y' = 0$$

$$\boxed{y' = \frac{-x}{y}}$$

6. At a sand and gravel plant, sand is falling off a conveyor and onto a conical pile at a rate of 10 cubic feet per minute. The diameter of the base of the cone is approximately three times the altitude. At what rate is the height of the pile changing when the pile is 6 feet high?



$$2r = 3h$$

$$r = \frac{3}{2}h$$

$$\frac{dV}{dt} = 10\ \text{ft}^3/\text{min}$$

$$\frac{dh}{dt} = ? \text{ when } h = 6\text{ft}$$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{3}{2}h\right)^2 \cdot h = \frac{1}{3}\cdot\frac{9\pi}{4}h^3$$

$$V = \frac{3\pi}{4}h^3$$

$$\frac{dV}{dt} = \frac{9\pi}{4}h^2 \cdot \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{\frac{dV}{dt}}{\frac{9\pi}{4}h^2} = \frac{4\cdot\frac{dV}{dt}}{9\pi h^2} = \frac{4\cdot 10}{9\pi\cdot 36} = \boxed{\frac{10}{81\pi}\ \text{ft/min}}$$

Review

$f(x) = -3x \tan x$

a. Find $f'(x)$.

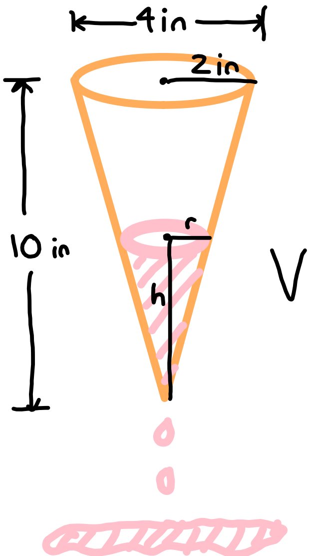
b. Find $f''(x)$.

(a) $f'(x) = -3\tan x - 3x\sec^2 x$

(b) $f''(x) = -3\sec^2 x - 3\sec^2 x - 3x \cdot 2\sec x \cdot \sec x \tan x$

$= -6\sec^2 x - 6x\sec^2 x \tan x$

1. A jumbo waffle cone from Sarah's Tasty Ice Cream Shoppe is 10 inches tall and has a 4 inch diameter at the top of the cone. Yesterday, my cone had a leak! Instead of eating it super fast, I decided to compare the rate of change of volume of ice cream to the rate of change of height of ice cream in the cone. How fast is the ice cream leaking out (in cubic inches per minute) when there are 5 inches of ice cream in the cone, if the height of ice cream in the cone is changing at a rate of 1 inch every 5 minutes?



$\frac{2}{10} = \frac{r}{h}$

$\frac{h}{5} = r$

$\frac{dh}{dt} = \frac{-1\text{ in}}{5\text{ min}}$

$\frac{dV}{dt} = ?$ when $h = 5\text{in}$

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{5}\right)^2 \cdot h$

$V = \frac{1}{3} \cdot \frac{\pi}{25} h^3$

$\frac{dV}{dt} = \frac{\pi}{25} h^2 \cdot \frac{dh}{dt}$

$= \frac{\pi}{25}(5\text{in})^2 \cdot \frac{-1\text{in}}{5\text{min}}$

$= \boxed{-\pi/5\ \text{in}^3/\text{min}}$

~~2. $x^3 + y^2 = 10$~~ $\quad x \arcsin y - y^2 = x^3 y$

a. Find y' in terms of x and y.

b. Find y'' in terms of x and y.

(a) $1 \cdot \arcsin y + \dfrac{xy'}{\sqrt{1-y^2}} - 2yy' = 3x^2 y + x^3 y'$

$\dfrac{xy'}{\sqrt{1-y^2}} - 2yy' - x^3 y' = 3x^2 y - \arcsin y$

$$\boxed{y' = \dfrac{3x^2 y - \arcsin y}{\dfrac{x}{\sqrt{1-y^2}} - 2y - x^3}}$$

(b) $$y'' = \frac{\left[\dfrac{x}{\sqrt{1-y^2}} - 2y - x^3\right]\left[6xy + 3x^2 y' - \dfrac{y'}{\sqrt{1-y^2}}\right] - \left[3x^2 y - \arcsin y\right]\left[(1-y^2)^{-1/2} + x \cdot \frac{1}{2}(1-y^2)^{-3/2}(-2yy') - 2y' - 3x^2\right]}{\left(\dfrac{x}{\sqrt{1-y^2}} - 2y - x^3\right)^2}$$

replace all y''s w/ answer from (a)

1. Locate the absolute extrema of the function on the closed interval. $f(x) = x^3 - \frac{3}{2}x^2$, $[-1, 2]$

$f'(x) = 3x^2 - 3x$

$3x(x-1) = 0$

$x = 0, 1 \rightarrow$ critical #'s

$f(-1) = -1 - \frac{3}{2} = -\frac{5}{2} \leftarrow$ absolute minimum

$f(0) = 0$

$f(1) = 1 - \frac{3}{2} = -\frac{1}{2}$

$f(2) = 8 - \frac{3}{2}(4) = 2 \leftarrow$ absolute maximum

2. Determine if Rolle's Theorem can be applied to f on the closed interval [a , b]. If Rolle's Theorem can be applied, find all values of c in the open interval (a, b) such that f'(c)=0.

$$f(x) = (x-3)(x+1)^2\ , \qquad [-1,3]$$

f cts on [-1,3] ✓
diff on (-1,3) ✓
f(-1) = f(3) ✓

Rolle's Theorem applies

$f(x) = (x-3)(x^2+2x+1)$

$= x^3+2x^2+x-3x^2-6x-3$

$= x^3-x^2-5x-3$

$f'(x) = 3x^2-2x-5$

$(3x-5)(x+1) = 0$

$\boxed{x = 5/3}$ $\quad \cancel{x = -1}$

3. Determine whether the Mean Value Theorem can be applied to f on the closed interval [a, b]. If the Mean Value Theorem can be applied, find all values of c in the open interval (a, b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$. $f(x) = x(x^2-x-2)$, $[-1,1]$

f(x) cts on [-1,1] ✓
diff on (-1,1) ✓

MVT applies

$$\frac{f(b)-f(a)}{b-a} = \frac{1(1-1-2)+(-1)(1-(-1)-2)}{1-(-1)}$$

$$= \frac{-2}{2} = -1$$

$f(x) = x^3-x^2-2x$

$f'(x) = 3x^2-2x-2$

$3x^2-2x-2 = -1$

$3x^2-2x-1 = 0$

$(3x+1)(x-1)$

$\boxed{x = -1/3}$, $\cancel{x = 1}$

Homework #6 (submitted Wed. 9/17)
2.5 # 1-39 odd; 43, 47 - Implicit Differentiation

Homework #7 (submitted Fri. 9/26)
2.6 # 15-23 odd - Related Rates
2.6 # 25, 27, 35 - Related Rates (more challenging problems)
3.1 # 17-31 odd - Absolute Extrema on an Interval

Homework #8 (due Fri. 10/3)
3.2 # 7-19 odd - Rolle's Theorem
3.2 # 31-37 odd - Mean Value Theorem
3.3 # 11-31 odd - Increasing, Decreasing, and Relative Extrema
3.4 #11-25 odd - Inflection Points and Concavity

Test #3 - Fri, 10/3