5. Find the open intervals on which the function is increasing or decreasing and locate all relative extrema. $y = \frac{x^2}{x^2-9}$

$$y' = \frac{(x^2-9)(2x) - x^2(2x)}{(x^2-9)^2} = \frac{2x^3-18x-2x^3}{(x^2-9)^2} =$$

$$= \frac{-18x}{\left[(x-3)(x+3)\right]^2}$$

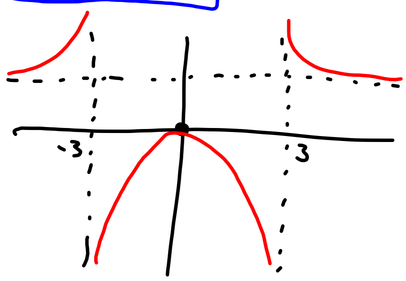
$\boxed{\text{critical \#'s: } -3, 0, 3}$

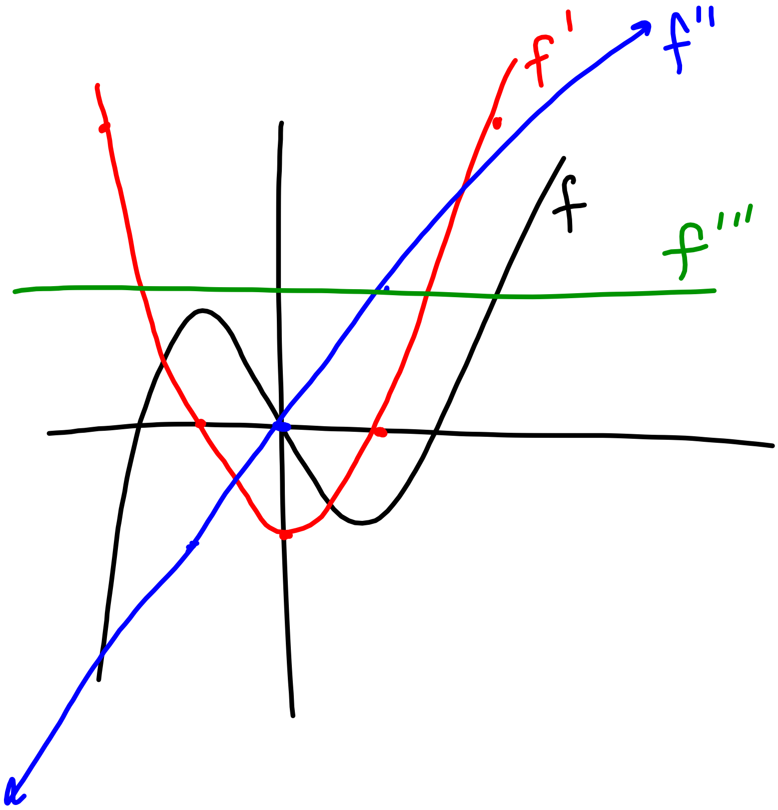
$f'(-4), f'(-1), f'(1), f'(4)$

$+ \quad -3 \quad + \quad 0 \quad - \quad 3 \quad -$

$\boxed{\begin{array}{l} f \text{ is increasing on } (-\infty,-3) \cup (-3,0) \\ f \text{ is decreasing on } (0,3) \cup (3,\infty) \end{array}}$

$\boxed{\begin{array}{l} f \text{ has a relative maximum @ } (0,0) \\ f \text{ has no relative minima} \end{array}}$





7. Find the points of inflection and discuss concavity of the graph of the function. $f(x) = \frac{x}{x^2+1}$

$$f'(x) = \frac{(x^2+1)(1) - x(2x)}{(x^2+1)^2} = \frac{x^2+1-2x^2}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$$

$$f''(x) = \frac{(x^2+1)^2(-2x) - (1-x^2)(2(x^2+1)\cdot 2x)}{(x^2+1)^4}$$

$$= \frac{\cancel{(x^2+1)}\cdot\left[(x^2+1)(-2x) - (1-x^2)(4x)\right]}{(x^2+1)^{\cancel{4}\,3}}$$

$$= \frac{-2x^3 - 2x - 4x + 4x^3}{(x^2+1)^3} = \frac{2x^3 - 6x}{(x^2+1)^3} = \frac{2x(x^2-3)}{(x^2+1)^3}$$

$$= \frac{2x(x-\sqrt{3})(x+\sqrt{3})}{(x^2+1)^3}$$

$f''(-2)$ $f''(-1)$ $f''(1)$ $f''(2)$

$-$ $-\sqrt{3}$ $+$ 0 $-$ $\sqrt{3}$ $+$

inflection points:

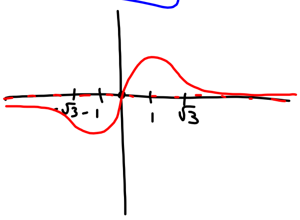
$\left(-\sqrt{3}, \frac{-\sqrt{3}}{4}\right)$

$(0, 0)$

$\left(\sqrt{3}, \frac{\sqrt{3}}{4}\right)$

f is concave up on $(-\sqrt{3}, 0) \cup (\sqrt{3}, \infty)$

f is concave down on $(-\infty, -\sqrt{3}) \cup (0, \sqrt{3})$



12. The radius of a right circular cylinder is given by $\sqrt{t+2}$ and its height is $\frac{1}{2}t$, where t is time in seconds and the dimensions are in inches. Find the rate of change of the volume with respect to time. Volume of a cylinder is given by $V = \pi r^2 h$, where r is the radius of the cylinder and h is the height.

$r = \sqrt{t+2}$

$h = \frac{1}{2}t$

$V = \pi r^2 h$

$V = \pi\left(\sqrt{t+2}\right)^2 \cdot \frac{1}{2}t$

$V = \pi(t+2)\cdot\frac{1}{2}t$

$V = \frac{\pi}{2}t^2 + \pi t$

$$\frac{dV}{dt} = \boxed{\pi t + \pi}$$

$a =$ acceleration due to gravity $= -32\ ft/s^2$

An object is dropped from a height of 1 mile.
(a) What is the object's velocity (in ft/s) when it hits the ground?
(b) What is the object's average velocity from time 2s to 4s?

(a) $v(t) = s'(t)$

$v(t) = -32t$

$v(18.17) = -32(18.17)$

$= \boxed{-581.44\ ft/s}$

$s(t) = \frac{1}{2}at^2 + v_0 t + s_0$

$s(t) = -16t^2 + 5280$

object hits ground when $s(t) = 0$

$-16t^2 + 5280 = 0$

$-16t^2 = -5280$

$t^2 = \frac{5280}{16}$

$t = \sqrt{\frac{5280}{16}} = 18.17s$

(b) $\frac{s(4) - s(2)}{4-2}$

$= \frac{-16(4)^2 + 5280 - (-16(2)^2 + 5280)}{2}$

$= \frac{-16(16) + 16(4)}{2}$

$= \frac{-16 \cdot \overset{2}{\cancel{4}}(4^{3} - 1)}{\cancel{2}} = \boxed{-96\ ft/s}$

14. The radius of a sphere is expanding at a rate of 3 centimeters per second. Find the rate of change of the volume of the ~~cube~~ sphere when the radius is 12 centimeters.

$\frac{dr}{dt} = 3\ cm/s$

$\frac{dV}{dt} = ?$ when $r = 12\ cm$

$V_{sphere} = \frac{4}{3}\pi r^3$

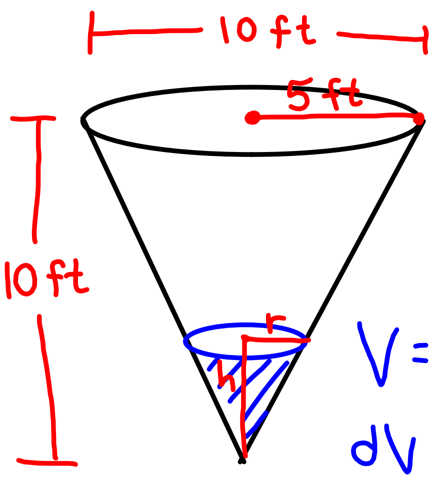
$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$

$= 4\pi(12)^2(3)$

$= \boxed{1728\pi\ cm^3/s}$

$$\begin{array}{r} 144 \\ \times\ 12 \\ \hline 288 \\ 1440 \\ \hline 1728 \end{array}$$

13. A conical tank is 10 feet across at the top and 10 feet deep. If it is being filled with water at a rate of 5 cubic feet per minute, find the rate of change of the depth of the water when it is 3 feet deep. The volume of a cone is given by $= \frac{1}{3}\pi r^2 h$, where r is the radius of the cone and h is the height. Give an exact answer in terms of π.



$$\frac{r}{h} = \frac{5}{10} \Rightarrow r = h/2$$

$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{dV}{dt} = 5\ ft^3/min$$

$$\frac{dh}{dt} = ? \text{ when } h = 3ft$$

$$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 \cdot h = \frac{1}{3}\cdot\frac{\pi}{4}h^3$$

$$\frac{dV}{dt} = \frac{\pi}{4}h^2 \cdot \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{\frac{dV}{dt}}{\frac{\pi}{4}h^2} = \frac{5\cdot 4}{\pi \cdot 3^2} = \boxed{\frac{20}{9\pi}\ ft/min}$$

Homework #6 (submitted Wed. 9/17)
2.5 # 1-39 odd; 43, 47 - Implicit Differentiation

Homework #7 (submitted Fri. 9/26)
2.6 # 15-23 odd - Related Rates
2.6 # 25, 27, 35 - Related Rates (more challenging problems)
3.1 # 17-31 odd - Absolute Extrema on an Interval

Homework #8 (due Fri. 10/3)
3.2 # 7-19 odd - Rolle's Theorem
3.2 # 31-37 odd - Mean Value Theorem
3.3 # 11-31 odd - Increasing, Decreasing, and Relative Extrema
3.4 #11-25 odd - Inflection Points and Concavity

Test #3 - Fri, 10/3